

Doebner-Goldin Equation for Electrodynamic Particle. The Implied Applications*

With Appendix: Dirac Equation for Electrodynamic Particles**

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Abstract. We set up the Maxwell's equations and subsequently the classical wave equations for the electromagnetic waves which together with the generating source, a traveling oscillatory charge of zero rest mass, comprise a particle traveling in the force field of an usual conservative potential and an additional frictional force f ; these further lead to a classical wave equation for the total wave of the particle. At the de Broglie wavelength scale and in the classic-velocity limit, the equation decomposes into a component equation describing the particle kinetic motion, which for $f = 0$ identifies with the usual linear Schrödinger equation as we showed previously. The f -dependent probability density presents generally an observable diffusion current of a real diffusion constant; this and the particle's usual quantum diffusion current as a whole are under adiabatic condition conserved and obey the Fokker-Planck equation. The corresponding extra, f -dependent term in the Hamiltonian operator identifies with that obtained by H.-D. Doebner and G.A. Goldin. The friction produces to the particle's wave amplitude a damping that can describe well the effect due to a radiation (de)polarization field, which is always by-produced by the particle's oscillatory charge in a (nonpolar) dielectric medium; such a friction and the resulting observable diffusion as intrinsically accompanying the particle motion was strikingly conjectured in the Doebner and Goldin original discussion. The radiation depolarization field in a dielectric vacuum has two separate significances: it participates to exert on another particle an attractive, depolarization radiation force which resembles in overall respects Newton's universal gravity as we showed earlier, and it exerts on the particle itself an attractive, self depolarization radiation force whose time rate gives directly the frictional force f .

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1. Introduction

While the usual linear Schrödinger equation has demonstrated to be adequate for the common nonrelativistic quantum systems, L. de Broglie suggested [1] in the 50s–60s that the quantum mechanical wave equation may be more generally nonlinear. Various forms of nonlinear equations have been proposed and investigated subsequently,

for a similar concern of internal states of particle as L. de Broglie's or from rather different fundamental considerations. Of these, the Doebner-Goldin form of nonlinear Schrödinger equation, Doebner-Goldin equation, represents a unique family which H.-D. Doebner and G.A. Goldin obtained in [2] by admitting observable diffusion current to the probability density of a quantum particle, and subjecting this to the continuity equation of a Fokker-Planck type on the basis of a unitary representation of an infinite-dimensional Lie algebra of vector fields and group of diffeomorphisms.

In view that it admits observable diffusion which commonly occurs to a greater or lesser degree in all macroscopic processes that are at the microscopic scale in majority cases executed by quantum particles, and in view of the physical significance associated with the unitary representation of group theory based on which it derives, the Doebner-Goldin equation can be anticipated to represent an important prediction of certain possible intrinsic processes accompanying quantum systems. What these possible processes may be has on the other hand remained as an open question prior to the present study. The main question may be formulated as that, what can be such an (intrinsic) dissipative process which disrupts not the stationary state of a quantum particle as an ordinary heat process would, and which in the meantime manifests itself an observable diffusion? From a measurement point of view at least such a process is viable even to a first degree, since such a process would not cause any detectable effect if a measurement is made over the damped probability density current and the damping in amplitude does not change with time. A theoretical recognition of such processes however would seem unrealistic until recently, in view that the mechanical nature of the quantum processes described by the usual Schrödinger equation had remained up to interpretation.

With overall experimental observations as input information the author recently proposed[3a-e] an internally electrodynamic (IED) model for simple particles such as electron, termed also basic particle formation (BPF) scheme in earlier reports [3a-h, j] (with coauthor P.-I. Johansson). The IED model, briefly, states that *a simple, single-charged particle is constituted of an oscillatory point charge q of a zero rest mass and the resulting electromagnetic waves propagating at the speed of light c* . In so far as the way the mechanism of the model operates, q can be of arbitrary quantity; q is to be given as an input (out of two sole input data, the other is the total energy of the charge) to yield the actual material particles. [For examples, of the elementary particles, for the isolatable charged one where the multiple- or "neutral-" charged particles are viewed as achieved by integration processes $n \leftarrow p + e + \nu_e$ and $N \leftarrow p + n$, clearly then $|q| = e$; for the, as of today, nonisolatable charged ones, quarks, $|q| = 1/3, 2/3$.] What form the basis of operating mechanism of the model merely are three elementary, experimentally firmly corroborated laws regarding electromagnetic waves: the Maxwell's equations in respect of wave propagation, the Doppler principle in respect of source motion effect, and the Planck energy equation in respect of energy discretization.

The obvious immediate motivation for proposing the IED particle model was to reconcile our understandings of particles with the various puzzling phenomena involving

particles to date. For example it was not understood that what are the internal, mechanical processes which causes a particle to manifest both as an extensive wave and a point object depending on methods of detection? And what are such internal mechanical processes which appear to simultaneously also command a particle to emit or absorb electromagnetic waves through the exchange of a portion or the whole of its own internal energy or inertial mass? What are the origin and nature of mass? In view of the uniform presence of charges in all of material particles, of the universality of the vacuum as a medium to as far as we know, and of the fact that in ordinary connection to charge the only pure waves propagating in this vacuum are electromagnetic waves, it is natural to expect that the internal processes of material particles are electromagnetic and that the operation of these hold the key to the answers to the various relevant puzzles.

In part as a broad test of the IED model, and in part as an endeavor of the understanding of a range of diverse phenomena from a common ground facilitated by it, a range of predictions of the fundamental properties of particles and relations have been made in terms "first-principles" solutions for the IED particle internal processes. Here, the "first-principles" refer to a minimal set of firmly established physics laws consisting mainly the few aforementioned; and the charge and the total (mechanical) energy, — corresponding to a characteristic oscillation frequency for a universal vacuum medium, are as two sole input data. The achieved predictions [3a-j] include: A particle has a spin and relativistic mass[3d-e] apart from the input characteristic charge and total energy; it is extensive as the result of its electromagnetic waves being extensive, and when traveling freely its waves evolve into a traveling, and in turn a standing, de Broglie phase wave between boundaries[3c,e]. As such, the particle obeys the de Broglie relations[3c,e]; its traveling de Broglie phase wave will produce constructive interference at integer times the de Broglie wavelength upon superposition of its different parts, and it in turn behaves like a point object owing to its charge, say, when scattering elastically with another point particle. More generally, in arbitrary potential fields under corresponding conditions the particle obeys the Schrödinger equation[3d,e] and the Dirac equation[3i]; the particle obeys the Einstein mass-energy relation[3c,e], the Galilean-Lorentz transformation[3f], and Newton's law of gravitation in attracting another particle[3j], among others.

It is natural that we extend in this paper the studies to aim to derive the Doebner-Goldin nonlinear Schrödinger equation, which we show will result when additionally subjecting the Schrödinger particle to a frictional force of the medium with the total system subjecting to an adiabatic condition. In part, this derivation provides an additional test of the IED basic-particle model. And in part, as the IED model itself suggests a natural origin of frictional force to be the radiation depolarization field always produced in a dielectric medium along with a particle's internal electromagnetic processes, the derivation and the solution will provide a formal elucidation for the connection of this with the Doebner-Goldin observable diffusion.

2. Particle model with electrodynamical internal processes

We consider an IED model particle is traveling at a velocity v as its oscillatory charge q does, for simplicity in a one-dimensional box along X -axis. The oscillation of the charge is associated with a total energy ε_q , ε_q being smallest at $v = 0$, $\varepsilon_q|_{v=0} = \mathcal{E}_q$. \mathcal{E}_q or ε_q may be endowed e.g. in a pair production in the vacuum. \mathcal{E}_q describes the ground state and therefore cannot be dissipated or detached from the charge except in a pair annihilation.

The charge will owing to its oscillation generate electromagnetic waves, with the radiation electric field \mathbf{E}^j and magnetic field \mathbf{B}^j (of the j th component) governed by the Maxwell's equations given in a medium of dielectric constant κ in zero external fields as

$$\nabla \cdot \mathbf{E}^j = \frac{\rho_q^j}{\epsilon}, \quad \nabla \cdot \mathbf{B}^j = 0, \quad \nabla \times \mathbf{B}^j = \mu \mathbf{j}_q^j + \frac{1}{c^2} \frac{\partial \mathbf{E}^j}{\partial t}, \quad \nabla \times \mathbf{E}^j = -\frac{\partial \mathbf{B}^j}{\partial t}. \quad (41)$$

Where ρ_q^j is the density and \mathbf{j}_q^j the current of the charge q of the particle, assuming no other charges and currents present; $\epsilon = \kappa \epsilon_0$ and $\mu \simeq \mu_0$, with ϵ_0 and μ_0 the permittivity and permeability of the vacuum, and c the velocity of light in the medium. (Until the specific application oriented denotations in Sec. 6, unless explicitly specified the respective media for conveying the particle and for the reference of measurement will not be explicitly specified in writing the electromagnetic variables.) Considering only regions sufficiently away from the source charge so that $\rho_q^j = j_q^j = 0$, making some otherwise standard algebra of the equations (41), and replacing the field variables by a more general dimensionless displacement φ^j , with $E^j = A\varphi^j$, $B^j = E^j/c = A\varphi^j/c$ and A a conversion constant, we obtain the corresponding classical wave equation for each component electromagnetic wave φ^j :

$$\frac{\partial^2 \varphi^j}{\partial T^2} = c^2 \nabla^2 \varphi^j \quad (42)$$

Here, in view of a Doppler effect to result from the source motion (to express explicitly later), we distinguish by the superscript j the component wave generated in the direction parallel with the source velocity v , denoted by $j = \dagger$, and the one in the direction antiparallel with v , denoted by $j = \ddagger$; within walls in stationary state there must also simultaneously prevail their reflected components and we may regard these as if being generated by a virtual charge (compared to the presently actual charge) which is reflected and traveling in $-X$ -direction, denoting by $j = \text{vir}\dagger$ and $j = \text{vir}\ddagger$. Apparently, the total wave given by the sum of all of the component waves, $\sum_j \varphi^j = \mathcal{Y}_0$, describes the particle.

Based on the general results of electrodynamics applied here to the particle's internal processes as governed by the basic equations (41)–(42), basic properties of a given particle can be predicted; in the remainder of this section we outline two directly relevant ones of these. The first is the total energy of the wave and accordingly the particle. As a general result of classical electrodynamics based on solution to the Maxwell's equations, the (41) here, combined with Lorentz force law, an electromagnetic

wave (j) transmits at the speed of light c a wave energy ε^j and a linear momentum $p^j = \varepsilon^j/c$. In virtue of the stochastic nature of the electromagnetic waves, its total dynamical quantities, the total wave energy and linear momentum here, are appropriately the geometric means,

$$\varepsilon = \sqrt{\varepsilon^\dagger \varepsilon^\ddagger}, \quad p = \sqrt{p^\dagger p^\ddagger}; \quad \sqrt{\varepsilon^\dagger \varepsilon^\ddagger} = \sqrt{p^\dagger p^\ddagger} c \quad \text{or } \varepsilon = pc. \quad (49)$$

From the underlining laws afore-used, mathematically the amplitudes of E^j, B^j, φ^j , etc, and accordingly ε^j, p^j, E and p are permitted to take on continuous values.

Following M. Planck's discovery of quantum theory in 1901, it has been additionally understood that the amplitudes of these quantities are *in nature* quantized; the total wave energy of an electromagnetic wave of frequency $\omega/2\pi$ is $\varepsilon = n\hbar\omega$, that is, ε consists in general of n momentum-space quanta, or photons, each of an energy $\hbar\omega$. The electromagnetic wave comprising our basic particle, like an electron, positron, etc., has, based on experimental indications especially the pair processes, a "single energy quantum", $n = 1$; the Planck energy equation for the total wave of the particle therefore is

$$\varepsilon = \hbar\omega. \quad (4)$$

This total wave of a single energy quantum here has in a one-dimensional box two components, φ^\dagger and φ^\ddagger , a situation no different from discussed after (42). Their wave frequencies are Doppler displaced to ω^\dagger and ω^\ddagger as a result of the source motion (to express explicitly below); and similarly as (4), $\omega^\dagger = \varepsilon^\dagger/\hbar$, $\omega^\ddagger = \varepsilon^\ddagger/\hbar$. Further from (49) we have $\omega = \sqrt{\omega^\dagger \omega^\ddagger}$. For the total wave comprising the particle, ε represents therefore the total energy of the particle. It has been proven and formally expressed especially through quantum electrodynamics that, the Maxwell's equations and naturally the derivative classical wave equation continue to hold; and the quantization of the fields and the wave energy etc. formally is the result of subjecting the corresponding canonical displacement and momentum, corresponding to the $u(=a\varphi)$ and \dot{u} here, to the quantum commutation relation $[u, \dot{u}] = i\hbar$. In this generalized framework, clearly the classical solution of a continuous amplitude for say ε merely is an approximation when n is large.

The second property is the inertial mass of the wave and thus the particle. The two components of the electromagnetic wave, (E^j, B^j) or φ^j , rapidly oscillating at a geometric mean frequency $\omega/2\pi$ and wavelength $\lambda = c/(\omega/2\pi)$, viewed at some distance and ignoring the detail of the oscillations will appear as if being two rigid objects, wavetrains, traveling at the speed of light c ; the two trains of the component waves together make a total wavetrain. In view that its speed of travel, c , is *finite* as contrasted to infinite, the total wavetrain has inevitably a *finite* inertia mass, denoting this by m . This mechanical representation of the total wave, as a rigid "wavetrain", permits us at once to express according to Newtonian mechanics the linear momentum of the wavetrain to be $p = mc$. Combining this with the classical electrodynamics result $\varepsilon + 0 = pc$ of (49) gives the kinetic energy of the wavetrain $\varepsilon = mc^2$; this is just the Einstein's mass-energy

relation (see e.g. [3c,g] for a detailed treatment). This energy and the Planck energy of (4) ought to equal of course, thus

$$m = \hbar\omega/c^2. \quad (\text{A.1})$$

The mass m of the total wavetrain comprising the particle naturally represents the mass of the particle, here acquired dynamically through the total motion of the waves of a geometric mean frequency $\omega/2\pi$. m is dependent on the particle velocity and is thus relativistic, see further after equation (14) below.

3. Wave equation of total motion of particle in external fields

To the particle we now apply a Coulomb force $F = -\nabla V$ owing to a conservative scalar potential V , and in addition a viscous force f ; these give a total applied force $F' = F + f$. We express the f as follows. Suppose out of the total oscillation of the charge, a fractional displacement u_q only produces radiation and is defined here to be equal to the wave displacement $u = a\mathcal{Y}$, \mathcal{Y} being the dimensionless total wave displacement in the field of applied potentials; this in zero applied potential field is the \mathcal{Y}_0 earlier. Making direct analogy to the viscous force of ordinary mechanics, we can write down the frictional force opposing the total motion of the particle as $f = \sum_n \frac{b_n}{(a\mathcal{Y})^n} \left(\frac{d(a\mathcal{Y})}{dT}\right)^n$ in units of N. That is, in general f is a function of the time rate of the total displacement of the charge or alternatively of the resulting wave displacement in the medium; section 7 will give a concrete representation of such a force. Assuming $d(a\mathcal{Y})/dT$ is small, so to a good approximation $f = \frac{b_1}{\mathcal{Y}L_\varphi} \frac{d\mathcal{Y}}{dT} = \frac{b_1}{\mathcal{Y}L_\varphi} \left(\frac{\partial\mathcal{Y}}{\partial T} + \nabla\mathcal{Y} \frac{\partial X}{\partial T}\right)$, where b_1 is a constant in units of Nms and is real; f is in units of N and is generally imaginary (pointed out by D. Schuch) for \mathcal{Y} being generally complex. This may rewrite as

$$f = -\frac{2mD}{\mathcal{Y}L_\varphi} \left(\frac{\partial\mathcal{Y}}{\partial T} + \frac{\partial\mathcal{Y}}{\partial X} W\right)$$

where

$$D = -\frac{b_1}{2m}, \quad (6)$$

$$W = -\frac{i\beta_1 \hbar v_{\text{obs}}}{2mD} - W^* = \frac{i\beta_1 \hbar (\nabla\mathcal{Y}^*)\mathcal{Y}}{2m|\mathcal{Y}|^2}. \quad (7)$$

$W (\equiv \frac{\partial X}{\partial T} = \partial\omega/\partial k)$ is the wave speed of \mathcal{Y} , and W^* ($= -\partial X/\partial T = \partial\omega'/\partial k'$) of that of the imaginary \mathcal{Y}^* . The expressions in (7) follow firstly from the requirement that W is in direct proportion with the velocity v_{obs} of the current $j_{\text{obs}} (= v_{\text{obs}}\rho)$ of the probability density $\rho (= |\mathcal{Y}|^2)$ in order to ensure the continuity of current in a non-absorbing medium. That is, $W = Iv_{\text{obs}} - W^*$, where the imaginary W^* is subtracted from the generally complex Iv_{obs} . The current $j_{\text{obs}} = v_{\text{obs}}\rho$ of ρ with a uniform translation at velocity v alternatively is according to Fick's law the diffusion of a varying ρ in a viscous medium, $j_{\text{obs}} = -D\nabla\rho$, with D the diffusion constant. So, $v_{\text{obs}} = \frac{j_{\text{obs}}}{\rho} = -\frac{D\nabla\rho}{\rho} = -\frac{D}{\rho}[(\nabla\mathcal{Y}^*)\mathcal{Y} + \mathcal{Y}^*\nabla\mathcal{Y}]$; here we have taken D to be as defined in

(6) and this will receive a justification later through the role of j_{obs} in (29). The above leads explicitly to the first and second expressions in (7) once we put the proportionality constant as $I = -\frac{i\beta_1\hbar}{2mD}$, where β_1 is a parameter yet to be determined [by the equation (29) below], and the other constants are inserted so that β_1 will have the simple solution value 1.

In virtue of the electrodynamic origin of F and inevitably also f which are empirically established for point particles, extending to the extensive IED particle here the two applied force and thus their total F' act apparently directly on the point charge. We now want to map this F' into a force directly interacting with the E^j, B^j or φ^j . We notice that by its mathematical form equation (42) represents just a classical wave equation for the electromagnetic wave φ^j , $a\varphi^j$ therefore a mechanical wave propagated in an elastic medium, and F' interacts with $a\varphi^j$ by an effective force F'_{med} acting on directly on this apparent medium. On the basis of this direct correspondence, but taken as a heuristic means only in this paper (so that we need not to firstly introduce with sufficient justifications at any detail the structure of this elastic medium), we shall below map the force F' into F'_{med} . Now, while F' drives the charge, of a mass m of the particle, into an acceleration $\partial^2\mathcal{Y}/\partial T^2$, F'_{med} drives the medium of mass \mathfrak{M}_φ (effectively) into acceleration $\partial^2\mathcal{Y}_{\text{med}}/\partial T^2$ in X -direction. Supposing the charge and the medium oscillate at a fixed phase difference if not in phase, the two accelerations must be equal, we thus have

$$F'_{\text{med}} = \frac{\mathfrak{M}_\varphi}{m}(F + f) = -\rho_l \left[\frac{V}{m} + \frac{2D}{\mathcal{Y}} \left(\frac{\partial \mathcal{Y}}{\partial T} + W \nabla \mathcal{Y} \right) \right]. \quad (8)$$

Where $\mathfrak{M}_\varphi = L_\varphi \rho_l$, with ρ_l the linear mass density of the medium along the wave path of a total effective length $L_\varphi = JL$ (\mathcal{Y} winds in J loops about the box side L).

We below further implement the force F'_{med} in wave equation (42) similarly using the heuristic approach by applying directly Newton's laws to the apparent elastic medium. In the apparent medium \mathcal{Y} corresponds to a physical, transverse displacement $u = a\mathcal{Y} = aC \sum_j \varphi^j$ as produced by the disturbance of the charge oscillation, with a a conversion constant of length dimension. The deformed elastic medium is consequently subject to a tensile force $F_R = \rho_l c^2$, with c the velocity of light at which φ^j propagates. This force F_R and the applied force F'_{med} together give the total force acting on the particle through directly acting on the apparent elastic medium

$$F'_R = F_R - F'_{\text{med}} = \rho_l \left[c^2 + \frac{V}{m} + \frac{2D}{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial T} + \frac{\beta_1 D i \hbar}{m |\mathcal{Y}|^2} |\nabla \mathcal{Y}|^2 \right], \quad (9)$$

where the minus sign in front of F'_{med} represents that this force tends to contract the chain. Consider on the linear chain of the medium a segment ΔL at $(X, X + \Delta X)$ is upon deformation tilted from its equilibrium ΔX an angle $\vartheta(X)$ and $\vartheta + \Delta\vartheta(X + \Delta X)$; assuming \mathcal{Y} is small, F_R will be uniform across the entire wave path L_φ . The transverse (Z -) component force acting on it is $\Delta F'_{Rt} = F'_R [\sin(\vartheta + \Delta\vartheta) - \sin \vartheta]$, with $[\sin(\vartheta + \Delta\vartheta) - \sin \vartheta] = [1 + O(\vartheta)] \Delta\vartheta \simeq \Delta\vartheta = \nabla^2(a\mathcal{Y}) \Delta R$; $O(\vartheta)$ collects the higher

order terms and is dropped (this term leads to anharmonicity and not damping, and as can be shown this in general leads not to a Doebner-Goldin form of nonlinear term). Substituting in the above with (9) for F'_R we have

$$\Delta F'_{Rt} = F'_R \nabla^2 (a\mathcal{Y}) \Delta R = a\rho_l \left[c^2 + \frac{V}{m} + \frac{2D}{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial T} + \frac{\beta_1 D i \hbar}{m\rho} |\nabla \mathcal{Y}|^2 \right] \nabla^2 \mathcal{Y} \Delta R. \quad (10)$$

Applying Newton's second law to the segment of a mass $\Delta \mathfrak{M}_\varphi = \rho_l \Delta L, \simeq \rho_l \Delta X$, we have $\rho_l \Delta X \frac{\partial^2 (a\mathcal{Y})}{\partial T^2} = \Delta F'_{Rt}$. Placing (10) in it, dividing $a\rho_l \Delta X$, we get the equation of motion for per unit length per unit mass density of the elastic chain of medium at X , or equivalently the classical wave equation for the total (electromagnetic) wave of the particle:

$$\frac{\partial^2 \mathcal{Y}}{\partial T^2} = \left[c^2 + \frac{V}{m} + \frac{2D}{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial T} + \frac{\beta_1 D i \hbar}{m\rho} |\nabla \mathcal{Y}|^2 \right] \nabla^2 \mathcal{Y}. \quad (11)$$

In summary, (11) has a basic part $\frac{\partial^2 \mathcal{Y}_0}{\partial T^2} = c^2 \nabla^2 \mathcal{Y}_0$ which one will get from summing over all j values the wave equations (42) given earlier directly from the Maxwell's equations in zero applied potential, and it has an additional part describing the effect of the applied force F'_{med} , derived with the help of the "heuristic elastic medium" approach.

Concerning the solution of (11), for the present we only consider explicitly the case of $D = 0$. So (11) reduces to $\frac{\partial^2 \mathcal{Y}_0}{\partial T^2} = [c^2 + \frac{V}{m}] \nabla^2 \mathcal{Y}_0$; this being linear, thus $\mathcal{Y}_0 = \sum_j \varphi^j$ and

$$\frac{\partial^2 \varphi^j}{\partial T^2} = \left[c^2 + \frac{V}{m} \right] \nabla^2 \varphi^j \quad (12)$$

Supposing also V is a constant, V_c , equation (12) can be immediately solved to consist of plane waves, $\varphi^\dagger = C \exp[i(k^\dagger X - \omega^\dagger T + \alpha_0)]$, $\varphi^\ddagger = -C \exp[i(k^\ddagger X + \omega^\ddagger T - \alpha_0)]$. Where $k^j = \gamma^j K$ are the Doppler-displaced wavevectors for the wave generated parallel with the source velocity v ($j = \dagger$) and antiparallel with v ($j = \ddagger$), and $\omega^j = \gamma^j \Omega$ are the corresponding angular frequencies, with $\gamma^\dagger = 1/(1 - v/c)$, $\gamma^\ddagger = 1/(1 + v/c)$. K and $\Omega = Kc$ are the values of k^j and ω^j at $v = 0$, c being the velocity of light as before.

The explicit superposition of the incident waves $\varphi^\dagger, \varphi^\ddagger$ and their reflected ones $\varphi^{\text{vir}\dagger}, \varphi^{\text{vir}\ddagger}$ give a standing wave (for a systematic representation see [3a-c,e]):

$$\mathcal{Y}_0 = \sum_j \varphi^j = C e^{i[(K+k_d)X]} e^{-i\omega T}, \quad (13)$$

where

$$k_d = \sqrt{(k^\dagger - K)(K - k^\ddagger)} = \gamma K_d, \quad K_d = \left(\frac{v}{c}\right) K; \quad \omega = \sqrt{\omega^\dagger \omega^\ddagger} = \gamma \Omega; \quad (14)$$

$\gamma = \sqrt{\gamma^\dagger \gamma^\ddagger} = 1/\sqrt{1 - v^2/c^2}$. Canceling ω between (14) and (A.1) gives further $m = \gamma M$, with $M = \hbar \Omega / c^2$ the classic-velocity limit ($v^2/c^2 \rightarrow 0$) of m , i.e. the rest mass of the particle. An explicit inspection of (13) will readily show that k_d is the de Broglie wavevector (for an existing elucidation see e.g. in [3c,i]), K_d being its value at the limit

$v^2/c^2 \rightarrow 0$. We shall later (see after equation 23) generalize the representation to the case where V may be arbitrarily varying in L ; until then we shall proceed the following discussion for the constant V , V_c . Substitution of \mathcal{Y}_θ in the total wave equation given from the linear sum of the wave equations (12) over all j , i.e. with $D = 0$ in (11), directly gives the expected relativistic energy-momentum relation for the particle[3b], which gives an additional check that \mathcal{Y}_θ of (13) is the correct solution to the total wave equation.

For the solution of wave equation (11) with D finite we shall use the trial function:

$$\mathcal{Y} = \mathcal{Z}\mathcal{Y}_\theta, \quad \text{where} \quad \mathcal{Z} = e^{iQ}, \quad Q = Q_1 + iQ_2. \quad (15)$$

\mathcal{Z} represents a damping factor; Q_1 and Q_2 are real variables and are in general functions of X, T . We shall restrict ourselves to the case where D is small and accordingly $|iQ| \ll |i(K + k_d)X - i\omega T|$. Under such a condition, for the derivation of a nonlinear Schrödinger equation in question below, until the context of equation (33), an explicit solution form of Q needs not be known.

4. Transformation to wave equation for kinetic motion of particle

At the classic-velocity limit $v^2/c^2 \rightarrow 0$, the total wave function \mathcal{Y}_θ reduces to (see [3a-c]) $\lim_{v^2/c^2 \rightarrow 0} \mathcal{Y}_\theta = Ce^{i(K_d X - \Omega_d T)}$ with $\Omega_d (= \frac{1}{2}\Omega(\frac{v}{c})^2) = \frac{1}{2}K_d v + V_c$, which is equivalent to the solution for Schrödinger equation for an identical system as described by the wave equation (11) in the case of $D = 0$ and $V = V_c$. Therefore, as we noted in [3a-c], equation (11) must inevitably have a direct correspondence to the Schrödinger equation, and the remaining question mainly then was to identify a physically justifiable procedure to transform (11) to a form of the Schrödinger equation. Such a formal procedure was elaborated in detail in [3a-c] by a back-substitution of the explicit function \mathcal{Y}_θ in wave equation (11) in the case of $D = 0$; by use of the Fourier theorem the procedure further led to a Schrödinger equation for V arbitrarily varying and also, by a straightforward extension, for three dimensions. For the present case of D being in general finite, we below similarly first reduce and simplify wave equation (11) at the classic-velocity limit $v^2/c^2 \rightarrow 0$ to a form such that the K - and K_d - processes are separable, by means of back-substitution of the formal function \mathcal{Y} (15) where the function \mathcal{Y}_θ is explicitly known and Q assumed small.

We first prepare for the separation of the K - and K_d -processes in three aspects, the first two being similar as for the case $D = 0$ [3a,b]: (i) We observe that (11) contains the derivative $\frac{\partial^2 \mathcal{Y}}{\partial T^2}$ which relates to the acceleration of the particle and, as such, the K - and K_d - processes are not separable; but the two processes are separable for the first derivative $\frac{\partial \mathcal{Y}}{\partial T}$ which relates to the total energy (for a detailed analysis see [3b]). This suggests us to lower the time derivative one order as $\frac{\partial^2 \mathcal{Y}}{\partial T^2} \simeq \frac{\partial}{\partial T}[(\frac{\partial \mathcal{Y}_\theta}{\partial T})e^{iQ} + \mathcal{Y}_\theta e^{iQ}i\frac{\partial Q}{\partial T}] \simeq \frac{\partial}{\partial T}[-i\omega\mathcal{Y} + 0] = -i\omega\frac{\partial \mathcal{Y}}{\partial T}$. (ii) In the two terms $\frac{V}{m}\nabla^2 \mathcal{Y}$ and $\frac{\beta_1 D i \hbar}{m \rho} |\nabla \mathcal{Y}|^2 \nabla^2 \mathcal{Y}$ in (11), the coefficients in front of $\nabla^2 \mathcal{Y}$, being approximately the scale of quadratic thermal velocity v^2 or lesser, are relatively small for V and D being small; and also these are constant. So

in these, consistent with the classic-velocity limit $v^2/c^2 \rightarrow 0$ in question, $\nabla^2 \mathcal{Y}$ can to good approximation be replaced by its computed value: $\nabla^2 \mathcal{Y} = \nabla[(\nabla \mathcal{Y}_0)e^{iQ} + \mathcal{Y}_0 e^{iQ} i \nabla Q] \simeq i(K + k_d)\nabla \mathcal{Y} + 0 = -(K^2 + k_d^2)\mathcal{Y}$, where in going to the second last expression we dropped the cross-term products between the mutually orthogonal ∇e^{iKX} and $\nabla e^{ik_d X}$ whose contribution to the final expectation value is in general zero (for an explicit proof see [3a,b]). Using the identity relation $\gamma^2 = 1 + \gamma^2 \frac{v^2}{c^2}$, the above rewrites $\nabla^2 \mathcal{Y} = -\gamma^2 K^2 \mathcal{Y}$. (iii) In the two terms $\frac{2D}{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial T} \nabla^2 \mathcal{Y}$ and $c^2 \nabla^2 \mathcal{Y}$ in (11), the coefficients $(\partial \mathcal{Y}/\partial T)/\mathcal{Y} \propto -i\omega$ and c^2 are large, with ω being the scale of the particle's total energy. So, in these the $\nabla^2 \mathcal{Y}$ ought to be kept in functional form. But in the first of the two terms the large $\partial \mathcal{Y}/\partial T$ itself effectively will be unaffected by the small V and D , and can therefore be replaced by its computed value $-i\omega \mathcal{Y}$ (used the small Q assumption), thus $\frac{2D}{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial T} = -i2D\omega$. Substituting with the reduced forms of (i)–(iii) for the respective $\frac{\partial^2 \mathcal{Y}}{\partial T^2}$, $\nabla^2 \mathcal{Y}$, and $\frac{2D}{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial T}$ in (11), simplifying using the basic relation $K^2 \gamma^2 c^2 = \omega^2$ and the relation $mc^2 = \hbar \omega$ given in (A.1), multiplying the resulting equation by $-\frac{\hbar}{\omega}$, (11) finally reduces to

$$i\hbar \frac{\partial \mathcal{Y}}{\partial T} = -\frac{\hbar^2}{m} \nabla^2 \mathcal{Y} + V_c \mathcal{Y} + i2D\hbar \nabla^2 \mathcal{Y} + i\beta_1 D\hbar \frac{|\nabla \mathcal{Y}|^2}{|\mathcal{Y}|^2} \mathcal{Y}. \quad (16)$$

We next proceed to separate in wave equation (16) the K - and the K_d - processes, which are inexplicitly contained in a factor γ in each term as we will see explicitly below, and based on this we further reduce the equation at the classic-velocity limit. To this end, with \mathcal{Y} formally given in (15), we first compute each derivative in (16) explicitly, and expand the γ factor ($\gamma = 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \frac{v^4}{c^4} + \dots$) in each:

$$\begin{aligned} \frac{\partial \mathcal{Y}}{\partial T} &= -i\gamma \Omega \mathcal{Y} = -i[\Omega + \Omega_d(1 + \frac{3}{4} \frac{v^2}{c^2} + \dots)]\mathcal{Y}, \quad \text{where } \Omega_d = \frac{1}{2}\Omega_d, \quad \Omega_d = \left(\frac{v}{c}\right)^2 \Omega, \\ \frac{1}{m} \nabla^2 \mathcal{Y} &= -\frac{\gamma^2 K^2 \mathcal{Y}}{\gamma M} = -\frac{\gamma K^2 \mathcal{Y}}{M} = \left[-\frac{K^2}{M} - \frac{K_d^2}{2M}(1 + \frac{3}{4} \frac{v^2}{c^2} + \dots)\right]\mathcal{Y}, \\ \nabla \mathcal{Y} &= i(K + \gamma K_d)\mathcal{Y}, \quad |\nabla \mathcal{Y}|^2 = (\nabla \mathcal{Y}^*)(\nabla \mathcal{Y}) = (K + \gamma K_d)^2 |\mathcal{Y}|^2. \end{aligned} \quad (17)$$

On equal footing as the above, \mathcal{Y} expands in its exponent as

$$\mathcal{Y} = C e^{i[(K + \gamma K_d)X - (\Omega + \Omega_d(1 + \frac{3}{4} \frac{v^2}{c^2} + \dots))T + Q]}. \quad (18)$$

The condition $v^2/c^2 \rightarrow 0$ in general ensures $K \gg K_d$, $\Omega \gg \Omega_d$. So, on the scales of K_d and Ω_d , the harmonic functions e^{iKX} and $e^{-i\Omega T}$ oscillate so rapidly that they present to any external observation effectively constants. Hence, $e^{-i\Omega T} \simeq 1$, $e^{iKX} \simeq 1$; and

$$\lim_{v^2/c^2 \rightarrow 0} \mathcal{Y} = C e^{i[K_d X - \Omega_d T + Q]} = \mathcal{Z} \Psi_0 \equiv \Psi, \quad \Psi_0 = C e^{i[K_d X - \Omega_d T]}. \quad (19)$$

Taking accordingly the classic-velocity limit of the relations of (17), substituting in the resulting relations with (19) for Ψ and its derivatives ($\nabla^2 \Psi = -K_d^2 \Psi$, $\frac{\partial \Psi}{\partial T} = -i\Omega_d \Psi$, $\nabla \Psi = iK_d \Psi$, $\nabla \Psi^* = -iK_d \Psi^*$, $|\nabla \Psi|^2 = K_d^2 |\Psi|^2$ for small Q assumption as earlier) for

the K_d -, Ω_d - terms while keeping the K -, Ω -terms as computed values which are large and will be unaffected for V and D being assumed to be relatively small, we have

$$\begin{aligned} \lim_{v^2/c^2 \rightarrow 0} \frac{\partial \mathcal{Y}}{\partial T} &= -i\Omega\Psi + \frac{\partial \Psi}{\partial T}, \quad \lim_{v^2/c^2 \rightarrow 0} \frac{\nabla^2 \mathcal{Y}}{m} = -\frac{K^2 \Psi}{M} + \frac{\nabla^2 \Psi}{2M}, \quad \lim_{v^2/c^2 \rightarrow 0} \nabla \mathcal{Y} = iK\Psi + \nabla \Psi, \\ \lim_{v^2/c^2 \rightarrow 0} \nabla \mathcal{Y}^* &= -iK\Psi + \nabla \Psi^*, \quad \lim_{v^2/c^2 \rightarrow 0} |\nabla \mathcal{Y}|^2 = K^2 |\Psi|^2 + |\nabla \Psi|^2. \end{aligned} \quad (20)$$

We dropped the cross-term products in the last relation of (20) for similar consideration as earlier. Finally, subjecting wave equation (16) to the classic-velocity limit and substituting in the resulting equation with the expressions of (20) we have

$$\begin{aligned} \hbar\Omega\Psi + i\hbar\frac{\partial \Psi}{\partial T} &= \frac{\hbar^2 K^2}{M}\Psi - \frac{\hbar^2}{2M}\nabla^2 \Psi + V_c\Psi - i2D\hbar K^2\Psi + iD\hbar\nabla^2 \Psi \\ &\quad + \frac{i\beta_1 D\hbar}{|\Psi|^2} [K^2 \gamma^2 |\Psi|^2 + |\Psi|^2 |\nabla \Psi|^2] \Psi. \end{aligned} \quad (21)$$

Equation (21) multiplied by $\frac{1}{\Psi}$ contains a component equation

$$\hbar\Omega = \frac{\hbar^2 K^2}{M} - i2D\hbar K^2 + i\beta_1 D\hbar K^2 \quad (22)$$

for a monochromatic electromagnetic wave produced by the given source but at zero velocity, and is not of our direct interest here. This equation holds always true for a given particle of a fixed rest mass and can be subtracted from $\frac{1}{\Psi} \times (21)$; multiplying Ψ back to the resulting equation from left, we obtain

$$i\hbar\frac{\partial \Psi}{\partial T} = -\frac{\hbar^2}{2M}\nabla^2 \Psi + V_c\Psi + iD\hbar\nabla^2 \Psi + i\beta_1 D\hbar \frac{|\nabla \Psi|^2}{\rho} \Psi. \quad (23)$$

If V varies arbitrarily with X , thus $V = V(X, T)$, φ^\dagger and φ^\ddagger are in general no longer plane waves. On the other hand, assuming $V(X, T)$ is well behaved, we can divide L into a large, N number of small divisions of width ΔX each. In each small division, $(X_j, X_j + \Delta X)$, the potential, $V(X_j, T) = V_{cj}$, continues to be approximately constant and is exactly so in the limit $\Delta X = 0$, and here the above plane wave method holds valid. Elsewhere, $V(X_j, T) = 0$. Going through therefore the foregoing procedure similarly for each division, j , with $j = 1, \dots, N$, we obtain equations of identical forms as (19), (23), etc., except with Ψ , K_d , Ω_d etc. denoted by $\Psi_{K_{dj}}$, K_{dj} , $\Omega_{dK_{dj}}$, etc. The $\{\Psi_{K_{dj}}(\mathbf{R}, T)\}$'s are mutually orthogonal and form a complete set. So the total wave function is the sum

$$\Psi(X, T) = \frac{1}{\sqrt{N}} \sum_{K_{dj}} A_{K_{dj}} \Psi_{K_{dj}}(X, T) = \frac{1}{\sqrt{N}} \sum_{K_{dj}} A_{K_{dj}} C e^{-i\sigma_{dj}T + iQ} \cdot e^{iK_{dj}R}; \quad (24)$$

or,

$$\Psi = \mathcal{Z}\Psi_0, \quad \Psi_0 = \xi e^{-i\sigma_d T}, \quad \mathcal{Z} = e^{iQ_1 - Q_2}, \quad \xi = \frac{1}{\sqrt{N}} \sum_{K_{dj}} A_{K_{dj}} C e^{iK_{dj}X - i(\sigma_{dj} - \sigma_d)T}, \quad (24)'$$

with $A_{K_{dj}} C e^{-i\sigma_{dj}T + iQ} = 2\pi \sum_{s=1}^N \Psi(X_s, T) e^{-iK_{dj} \cdot X_s}$ the Fourier transform of $\Psi(X_s, T)$.

Multiplying $\frac{1}{\sqrt{N}}A_{\kappa_{dj}}$ through the corresponding equation of (23) for Ψ_j , summing the equations over all j values we have

$$\begin{aligned} i\hbar \frac{\partial \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j}{\partial T} = & -\frac{\hbar^2}{2M} \nabla^2 \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j + \sum_j V_{cj} \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j \\ & + iD\hbar \nabla^2 \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j + i\beta_1 D\hbar \frac{(\nabla \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j)^* (\nabla \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j)}{(\frac{1}{\sqrt{N}})^2 \sum_j A_{\kappa_{dj}}^2 \rho_j} \frac{1}{\sqrt{N}} \sum_j A_{\kappa_{dj}} \Psi_j. \end{aligned} \quad (25)$$

Where, $\sum_j V_{cj} = \dots + 0 \cdot V(X_{j-1}, T) + 1 \cdot V(X_j, T) + 0 \cdot V(X_{j+1}, T) + \dots = V(X_j, T)$; $A_{\kappa_{dj}}^* = A_{\kappa_{dj}}$ since the amplitude of the physical displacement Ψ must be real; $\rho = \sum_j \sum_j A_{\kappa_{dj}}^2 \rho_j = \sum_j A_{\kappa_{dj}}^* \Psi_j^* \sum_{j'} A_{\kappa_{dj'}} \Psi_{j'}$ for Ψ_j^* and $\Psi_{j'}$ mutually orthogonal and $A_{\kappa_{dj}}^*$ real; and $\sum_j \sum_j |\nabla A_{\kappa_{dj}} \Psi_j|^2 = \nabla \sum_j (A_{\kappa_{dj}} \Psi_j)^* \nabla \sum_j A_{\kappa_{dj}} \Psi_j$ for the two factors mutually orthogonal.

Substituting (24) in (25) we obtain a generalized result of (23), a wave equation describing the kinetic motion of the particle in an arbitrarily varying, well-behaved potential V :

$$i\hbar \frac{\partial \Psi}{\partial T} = -\frac{\hbar^2}{2M} \nabla^2 \Psi + V\Psi + iD\hbar \nabla^2 \Psi + i\beta_1 D\hbar \frac{|\nabla \Psi|^2}{\rho} \Psi. \quad (26)$$

Equation (26) is seen to represent an ordinary Schrödinger equation except for the extra, nonlinear term $iD\hbar \nabla^2 \Psi + i\beta_1 D\hbar \frac{|\nabla \Psi|^2}{\rho} \Psi$ due directly to the frictional force f .

5. Diffusion currents. Continuity equation. Doebner-Goldin Equation

Making some standard algebra to equation (26) and its complex counterpart leads to an equation for the total current $j_{tot} = j_{qm} + j_{obs}$ of the probability density $\rho = |\Psi|^2$:

$$\frac{\partial \rho}{\partial T} + \nabla(j_{qm} + j_{obs}) + (\beta_1 - 1)2D \frac{|\nabla \Psi|^2}{\rho} |\Psi|^2 = 0. \quad (27)$$

Where

$$j_{qm} = \frac{\hbar}{2Mi} [(\nabla \Psi^*) \Psi - \Psi^* \nabla \Psi], \quad j_{obs} = -D \nabla \rho = \frac{b_1}{2M} [(\nabla \Psi^*) \Psi + \Psi^* \nabla \Psi] \quad (28)$$

with j_{qm} the usual quantum diffusion current and j_{obs} the observable diffusion current as earlier except now expressed in terms of the classic-velocity limit function Ψ . The first quantity, j_{qm} , has an imaginary diffusion constant $D_{qm} = \frac{i\hbar}{2M}$ and this we know is to an external observer non-observable.

Suppose there are no "sinks" in the medium nor external reservoir in contact to the medium that trap or conduct the total current j_{tot} . So the particle and the (continuous) medium as a whole is adiabatic—a condition having an equal footing with the "unitary representation of vector field (of diffeomorphisms group)" employed in [2].

Then, equation (27) for the total probability density current of the particle, of a wave function Ψ governed by wave equation (26), needs to conform to the continuity equation which, for D being real and j_{obs} being observable, is of the Fokker-Planck type:

$$\frac{\partial \rho}{\partial T} + \nabla(j_{\text{qm}} + j_{\text{obs}}) = 0. \quad (29)$$

Comparison of this with (29) suggests the third term in (27) must vanish; so $\beta_1 = 1$.

With the β_1 value in (7) we find: $W + W^* = \frac{-i\hbar v_{\text{obs}}}{2mD}$, $W = +\frac{i\hbar}{2m\rho}(\nabla\Psi^*)\Psi$, and $W^* = +\frac{i\hbar}{2m\rho}(\nabla\Psi)\Psi^*$. With the β_1 value in turn directly in wave equation (26), we finally have

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\Psi + V\Psi + iD\hbar\nabla^2\Psi + iD\hbar\left(\frac{|\nabla\Psi|^2}{|\Psi|^2}\right)\Psi, \quad (30)$$

or

$$i\hbar\frac{\partial\Psi}{\partial t} = H'\Psi, \quad H' = H + iD\hbar G, \quad H = -\frac{\hbar^2}{2M}\nabla^2 + V, \quad G = \nabla^2\Psi + \left(\frac{|\nabla\Psi|^2}{|\Psi|^2}\right). \quad (30a)$$

Equation (30) is seen to be exactly the Doebner-Goldin form of nonlinear Schrödinger equation, the Doebner-Goldin equation, introduced in [2]. In view of their respective meanings, the unitary representation of vector fields in [2] and the probability density conservation in an adiabatic total system here are apparently two alternative but equivalent conditions. It is thus natural that the use of the latter here has led to the same result as based on the former in [2].

As was well appreciated in [2] (1994), the nonlinear total Hamiltonian H' as of (30a) is in general complex and not Hermitian as would be required by the usual linear Schrödinger equation; a complex nonhermitian Hamiltonian is today a topic of increasingly many studies. In this regard the foregoing derivation of equation (30) based on the IED particle model additionally points to that, underlining the complex H' and its imaginary part $iD\hbar G$ respectively are a complex total force $F'_{\text{med}} = F_{\text{med}} + i|f_{\text{med}}|$ and an imaginary frictional force f_{med} , a property drawn the author's attention by D. Schuch at the SNMP conference, Kiev, 2007. As is suggested by the mathematical form, we may comprehend the imaginary f_{med} as a physical variable orthogonal to the real F_{med} . As such, a measurement of the total force F'_{med} would then inform the modulus of it, $|F'_{\text{med}}| = \sqrt{F_{\text{med}}^2 + f_{\text{med}}^2}$, and not the direct addition of two scalar component forces and also not an ordinary vector sum $\mathbf{F}'_{\text{med}} = \mathbf{F}_{\text{med}} + \mathbf{f}_{\text{med}}$.

Concerning the solution for the Doebner-Goldin equation (30) we shall later only refer to an interesting and also relevant case treated by H.-D. Doebner and G.A. Goldin in [2]. Starting with the denotations specified in (24)' we have the more general expressions: $\rho = |\xi|^2 e^{-2Q_2}$, $\frac{\partial \rho}{\partial T} = -2|\xi|^2 e^{-2Q_2} \frac{\partial Q_2}{\partial T}$, $j_{\text{obs}} = \frac{b_1}{2M}[(\nabla\xi^*)\xi + \xi^*\nabla\xi + 2(\nabla Q_2)|\xi|^2]e^{-2Q_2}$, $j_{\text{qm}} = \frac{\hbar}{2Mi}[(\nabla\xi^*)\xi - \xi^*\nabla\xi - 2i(\nabla Q_1)|\xi|^2]e^{-2Q_2}$. Following [2] we put $Q_2 = 0$; the foregoing then become:

$$\rho = |\xi|^2, \quad \frac{\partial \rho}{\partial T} = \frac{\partial |\xi|^2}{\partial T}; \quad (31)$$

$$j_{\text{qm}} = \frac{\hbar}{2Mi}[(\nabla\xi^*)\xi - \xi^*\nabla\xi - 2i(\nabla Q_1)|\xi|^2], \quad j_{\text{obs}} = \frac{b_1}{2M}[(\nabla\xi^*)\xi + \xi^*\nabla\xi]. \quad (32)$$

Secondly, suppose the system is in stationary state, of which one of two possible descriptions is $\frac{\partial \rho}{\partial T} = 0$. (Since for $Q_2 = 0$, ρ of (31) does not contain D explicitly, so the "stationary state" here is not a small- D approximation but is exact as long as (30) holds. But we derived (30) based on a small D condition, which agrees with the small D requirement in [2].) Then $\rho = |\xi|^2$ of (31) is independent of time. Combining this with (27) follows $\nabla(j_{\text{qm}} + j_{\text{obs}}) = 0$. Or, $j_{\text{qm}} = D\nabla\rho + B$ with B a constant. Substituting in this last equation with (32) for j_{qm} , restricting $\nabla[(\nabla\xi^*)\xi - \xi^*(\nabla\xi)] = B$ to be independent of time which ensures that when $D = 0$, ξ is a solution to the usual Schrödinger equation, and further with the specific choice $B = 0$, one gets $\frac{-\hbar}{2Mi}(-2i\rho\nabla Q_1) = -D\nabla\rho$. Or, $\nabla Q_1 = -\frac{\Gamma\nabla\rho}{\rho}$ where $\Gamma = mD/\hbar$. Integrating gives: $Q_1 = -\Gamma \ln |\xi|^2$. Substituting in (24)' with this solution for Q_1 and the $Q_2 = 0$ earlier, one gets:

$$\Psi = \mathcal{Z}\Psi_0, \quad \Psi_0 = \xi e^{-i\mathcal{T}_a T}, \quad \mathcal{Z} = e^{-i\Gamma \ln |\xi|^2}. \quad (33)$$

Paper [2] also discussed that the other of the two possible descriptions of the stationary state to be $j_{\text{qm}} = 0$. This will also find a significant application in the examples later.

Different forms of the nonlinear term would imply other boundary conditions than an adiabatic one, or other applied forces than of the form here. In recent years, in terms of group theoretical approach H.-D. Doebner and G.A. Goldin [2] (1994) and A.G. Nikitin and A.G. Nikitin and R.O. Popovych [6] gave classifications of nonlinear Schrödinger equations in association with diffeomorphism group representations and in general terms. D. Schuch gave an insightful review [7] on the nonlinear Schrödinger equations proposed by different authors with analysis regarding the quantum physical justifiability of solutions, and introduced an interesting logarithmic form of nonlinear Schrödinger equation. H.-D. Doebner, A. Kopp and R. Zhdanov generalized in [6] nonlinearity to Dirac systems. Corresponding representations of these and beyond based on the IED particle model in the future can be similarly of value for test of the model and for gaining insight into the corresponding mechanical nature of nonlinearity of quantum systems.

6. Damping in dielectric media as generic application of the Doebner-Goldin equation

In most applications the motion of a macroscopic object will in general be dissipated or more restrictively, damped, to a greater and lesser degree. The dissipation is typically known in the form of heat exchange with the environment and manifesting as an observable diffusion current. But such a description for a single quantum particle system, as described by (30) being in stationary state, needs be taken in an effective, average way only. This directly follows from the circumstances that heat reflects in general an energy current composed of many random collisions of a large population of individual (quantum) particles, during which the particles in general deviate from stationary state. Apart from its possible applications in the aforesaid effective way, it

has been desirable[2, 4] to know whether there may exist a Doebner-Goldin form of observable diffusion accompanying a stationary-state quantum particle literally and as an intrinsic process. The IED particle model underlining the foregoing derivation of the Doebner-Goldin equation implies in fact the presence of such processes applicable essentially to all quantum particles in any dielectric media; we elucidate these below.

Consider an IED particle is moving in the total medium of an ordinary material medium n and the penetrating vacuum taken here literally to be dielectric, of a total dielectric constant κ as measured against a true empty space, the space after removal of the dielectric vacuum. An explicit knowledge of the structure of the dielectric vacuum[‡] is not needed for the dielectric relations given in this paper. The total medium and the particle are as a whole evidently adiabatic. Measured against the true empty space, the particle's component radiation electric field propagated in the total dielectric medium is E , and would be E_0 if "propagated"[8] in the empty space. (In this section we shall for conciseness drop the superscript j , either because this is not directly of concern or the variables actually may represent the geometric mean quantities.) When measured in the usual way against the vacuum with the vacuum regarded as effectively "non dielectric", the field E in the total medium would be E^0 ; E and E^0 represent the same force (note that the charge involved apparently causes no effect, for an original discussion see [31 e, g]) acting on the same medium as measured in the same inertial frame and must therefore be equal, $E \equiv E^0$; this is irrespective of against which medium the force is measured and represented. Supposing for simplicity the material medium is nonpolar, with the vacuum being naturally nonpolar, so the total dielectric medium is nonpolar and the charge produces in it a depolarization field E_p . The corresponding dimensionless wave displacements accordingly are: $\mathcal{Y}(\equiv \mathcal{Y}^0) = E/A$, $\mathcal{Y}_0 = E_0/A$, and $\mathcal{P} = E_p/A$.

Applying the standard dielectric theory for ordinary materials to the generalized dielectric system of an ordinary material and the vacuum here we can write down the following relations (for a systematic treatment see [3g,e]): $E(\equiv E^0 = \frac{E_0^0}{\kappa_n^0}) = \frac{E_0}{\kappa}$, $E_p = -\chi E$, $E = E_0 + E_p$, with

$$\kappa = \kappa_n^0 \kappa_0, \quad \chi = \kappa - 1 = (\chi_n^0 + 1)(\chi_0 + 1) - 1. \quad (34)$$

Where, χ is the susceptibility of the total dielectric medium and χ_0 that of the pure dielectric vacuum each measured against the empty space; κ_n^0 is the dielectric constant and χ_n^0 the susceptibility of the ordinary material medium n measured in the usual way against a "non-dielectric" vacuum; ϵ_0 , ($\equiv \epsilon_0^0$) = $\kappa_0 \epsilon_0$, is the permittivity of vacuum and ϵ_0 the permittivity of the empty space. Multiplying by $1/A$, taking the classic-velocity limit as in (18), the dielectric relations for the fields in the above become then

$$\Psi = \Psi_0/\kappa, \quad \mathcal{P} = -\chi\Psi, \quad \Psi = \Psi_0 + \mathcal{P}. \quad (35)$$

[‡] There exist today various propositions for the contents and structure of the vacuum as held in different fields like in QED, QCD, etc., or by individual authors including the "vacuonic vacuum structure" proposed by the present author [3g-h,e]; there appears to exist no direct experimental information regarding the explicit structure of the vacuum.

Comparing the first relation in the above with the general form of wave function for the Doebner-Goldin equation, $\Psi = \mathcal{Z}\Psi_\emptyset$ of (19) or more generally (24)', we have

$$\mathcal{Z} = 1/\kappa. \quad (36)$$

This states that, *the damping factor \mathcal{Z} corresponds rather generally to the inverse of the dielectric constant κ of the medium in which the particle resides.* In the case where ρ is independent of time, as specified by the $Q_2 = 0$ and small D conditions, Ψ and \mathcal{P} are described by the specific solutions (33), which combined with (36) gives the corresponding expressions for the two dielectric parameters

$$\kappa = 1/\mathcal{Z} = e^{i\Gamma \ln |\xi|^2}, \quad \chi = e^{i\Gamma \ln |\xi|^2} - 1. \quad (37)$$

In the specific case when no ordinary material presents, we have a pure dielectric vacuum, thus $\kappa_n^0 = 1$, $\chi_n^0 = 0$; (34) and (35) reduce to $\kappa = \kappa_0$, $\chi = \chi_0 = \kappa_0 - 1$, $\Psi_0 = \frac{\Psi_\emptyset}{\kappa_0} = \Psi_\emptyset + \mathcal{P}_0$, $\mathcal{P}_0 = -\chi_0\Psi_0$; and (36) and (37) reduce to $\mathcal{Z}_0 = 1/\kappa_0$. \mathcal{Z}_0 and κ_0 are for a specified particle here evidently universal constants, given that the vacuum is ubiquitous, isotropic and uniform throughout the space to as far as we know all of the time. As a consequence, the wave function Ψ_\emptyset appears to have never directly manifested itself in our present day's detections which are commonly based on the variation of the wave amplitude of a particle as a function of location and time; our only direct knowledge of the particle wave appears to be the Ψ_0 ($\equiv \Psi_\emptyset^0$) of which the \mathcal{Z}_0 or κ_0 is an inseparable component. Despite this, we see that first of all there presents a complete agreement between the prediction from the Doebner-Goldin equation, applied to the IED particle, that the electromagnetic waves "inside" (or comprising) a particle can in general admit damping but without decaying with time in amplitude, and the fact that electromagnetic waves "outside" (i.e. detached from) a particle, becoming directly observable, factually essentially do not decay with time in amplitude in the vacuum.

Further, the dielectric vacuum, hence the E_p^j field of a charge in it and accordingly the Ψ_\emptyset wave in the empty space, has an indirect yet profound manifestation according to a recent theoretical prediction [3j,f] by the author with coauthors. Namely, the E_p field participates to produce an attractive depolarization radiation force acted universally between two particles 1, 2 of masses M_1 and M_2 and charges q_1 and q_2 , separated at a distance R . This force is as the result of the Lorentz force in their mutual E_p , $B_0(\equiv B^0)$ fields say in the case of a pure vacuum: $F_{ii'} = q_j \frac{\Delta T q_{i'} E_{pi} B_{0i}}{M_{i'}}$, $i, i' = 1, 2$. The geometric mean of the mutual forces is $F_g = \sqrt{|\langle F_{12} \rangle \langle F_{21} \rangle|} = \frac{CM_1 M_2}{R^2}$, where $\langle \rangle$ represents time average, $|q_i|, |q_{i'}| = e$, $C = \pi \chi_0 e^4 / \epsilon_0^2 h^2 \rho_l$, e is the elementary charge and the other constants are as specified earlier; this force F_g was elucidated in [3j,f] to resemble in all respects Newton's universal gravity. To this application of the E_p field, the present study adds that the E_{pi}^j field of particle i producing the depolarization radiation force leads directly to a damping *in the particle's Schrödinger wave* $\Psi_{\emptyset i}$, by a factor \mathcal{Z}_0 , and the associated extra Hamiltonian term is a Doebner-Goldin nonlinear term added to that of the usual Schrödinger equation. In Sec. 7 we shall explicitly express the force directly

responsible for the damping. Also in this context, the $j_{\text{qm}} = 0$ solution mentioned earlier may be a case where the particle wave and thus j_{qm} is shielded, say by a material wall. And on the other side of the wall $j_{\text{qm}} = 0$; here only the Doebner-Goldin observable diffusion current j_{im} prevails. This directly corresponds to the property of the gravity which can not be shielded by any materials and on the other side of the wall as here it will propagate alone.

In another specific case when an ordinary dielectric medium presents and we represent the vacuum in the usual way as non-dielectric which thus effectively plays the role of an empty space in the dielectric relations, the total dielectric medium thus reduces to the ordinary dielectric material medium alone, thus $\kappa_0 = 1$, $\chi_0 = 0$. The relations of (34)–(35) now reduce to $\kappa = \kappa_n^0$, $\kappa_n^0 - 1 = \chi_n^0$, $\Psi_n = \Psi_0/\kappa_n^0$, etc., and (36) reduces to $1/\kappa_n^0 = \mathcal{Z}_n^0$. Finally, with $\rho^0 = |\xi^0|^2$, (37) reduces to $\kappa_n^0 = 1/\mathcal{Z}_n^0 = e^{i\Gamma \ln |\xi^0|^2}$, $\chi_n^0 = e^{i\Gamma \ln |\xi^0|^2} - 1$. That is, \mathcal{Z}_n^0 represents now a damping of the wave function Ψ_0 one would measure in a pure vacuum medium, into Ψ_n one will measure in the ordinary material medium of a dielectric constant κ_n^0 . This is an usual representation of a particle in a material medium; this is directly comparable to the familiar fact that a radiated (detached) electromagnetic wave in an optical material in general experiences a complex dielectric constant and susceptibility.

The above two specific situations would in general simultaneously enter in our material world, where a material particle commonly is more or less surrounded by other material particles or substances and which, in the extreme case when all ordinary material substances are absent, is left to be the dielectric vacuum itself. Besides, a material particle is electromagnetic in nature (in the sense that such a particle invariably contains an electric charge), which is a direct observational fact and is irrespective of the specific IED particle model employed in this study (whereas the IED particle model only is essential in leading to a formal relationship between the wave function, the electromagnetic field of the charge and the corresponding depolarization field of the particle). These two universality features of the material world determine that a depolarization (radiation) field presents intrinsically universally with a particle. It therefore follows that a Doebner-Goldin damping, identified here with a depolarization radiation field, is an intrinsic phenomenon presenting always to a material particle. Such a prospect that the nonlinear process could be intrinsically universally accompanied with the particle process as elucidated in the above two examples was strikingly conjectured in the Doebner-Goldin original paper[2].

7. Self depolarization radiation force: Gravity from the dielectric medium

We now give a concrete expression for the frictional force due to a total dielectric medium, of dielectric constant κ , against a particle moving in it. In the medium the particle's component radiation fields are $E^j, B^j (= E^j/c)$, $j = \dagger$ for the fields propagated in the direction parallel with v and $j = \ddagger$ for fields antiparallel with v similarly as earlier. These fields are the results after damping in the medium, from the un-damped $E_\emptyset^j, B_\emptyset^j$

measured in empty space, by a depolarization radiation field $E_p^j = -(E_0^j - E^j) = -\chi E^j$ and a corresponding $B_p^j = -(B_0^j - B^j) = -\chi_m B^j$ due to the presence of the medium, where $\chi_m = \sqrt{\kappa}^3 - 1$, and $c = c_0/\sqrt{\kappa}$ (the permeability is assumed to be 1 here). The charge, due to the E_p^j, B_p^j - fields induced by itself, is acted by a magnetic force according to the Lorentz force law:

$$\mathbf{F}_{m.p}^j = q\mathbf{v}_p^j \times \mathbf{B}_p^j = (\text{sign}) \frac{\chi\chi_m q^2 E^{j2}}{Mc} \hat{X}, \quad (38)$$

where $\mathbf{v}_p = qE_p/M$; $\text{sign} = +$ for $j = \dagger$ and $= -$ for $j = \ddagger$. Equation (38) expresses that, irrespective of the sign of the charge and of the momentary directions of the alternating fields generated by the charge to its right ($E^\dagger, \pm B^\dagger$ with a velocity c) and to its left ($E^\ddagger, \mp B^\ddagger$ with a velocity $-c$), $\mathbf{F}_{m.p}^j$ is always a *pull* to the charge on either side from the medium. In this connection, $\mathbf{F}_{m.p}$ refers to a self depolarization radiation force on the particle; and this represents a gravitational force on the particle from the dielectric medium.

While the particle is in stationary state, its (oscillatory) charge is constantly traveling in an alternating $+X$ - and $-X$ - directions. The charge when traveling in the $-X$ - direction is similarly acted by a pull on either side, but now the "sign" $= +$ for $j = \ddagger$ (wave generated opposite to the charge motion direction) and "sign" $= -$ for $j = \dagger$. So, on average the charge is acted from either side by a pulling force, an attraction, given by the geometric mean of the two Doppler-displaced forces:

$$\mathbf{F}_{m.p} = \sqrt{\mathbf{F}_{m.p}^\dagger \mathbf{F}_{m.p}^\ddagger} = (\text{sign}) \frac{\chi\chi_m q^2 E^2}{Mc} \hat{X}, \quad (39)$$

where $E = \sqrt{E^\dagger E^\ddagger}$. This mapped to the medium is similarly a pull to the deformed segment in question by its surrounding in the medium, corresponding to a reduced displacement of the medium, this is opposite to the tensile force associated in general with the usual E^j field.

We are here mainly interested in the resistance produced by $F_{m.p}$ against the total motion of the particle, that is the time rate of $F_{m.p}$ as measured over a certain time interval ΔT : $\int_0^{\Delta T} (dF_{m.p}/dT) dT \simeq \Delta T dF_{m.p}/dT$. With this, the frictional force following the usual definition is:

$$\begin{aligned} f_{m.p} &= \frac{\Delta T}{\mathcal{Y}} \frac{dF_{m.p}}{dT} = \frac{\Delta T}{(E/A)} \left(\frac{\partial F_{m.p}}{\partial T} + \frac{\partial F_{m.p}}{\partial X} \frac{\partial X}{\partial T} \right) \\ &= \frac{\Delta T \chi \chi_m q^2 A^2}{Mc} \left(\frac{\partial \mathcal{Y}}{\partial T} + \frac{\partial \mathcal{Y}}{\partial X} W \right) \end{aligned} \quad (40)$$

where $E = A\mathcal{Y}$ and $W = \partial X/\partial T$ as earlier. Similarly as $F_{m.p}$, $f_{m.p}$ is always opposite in direction to the tensile force F_R . We see that indeed, in both its acting as a resistance against the particle total motion and in its functional form, $f_{m.p}$ of (40) resembles fully the frictional force f expressed formally in (8) earlier.

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- [8] Mechanical wave can not be propagated in empty space. Here the " $E_{\emptyset*}$ propagated in empty space" should be understood as the field E propagated in the dielectric vacuum but after compensated for the damping in amplitude by the dielectric medium.

Appendix I

Dirac Equation for Electrodynamic Particles

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Abstract. We set up the Maxwell's equations and subsequently the classical wave equations for the electromagnetic waves which together with their generating source, an oscillatory charge of zero rest mass in general travelling, make up a particle travelling similarly as the source at velocity v in the field of an external scalar and vector potentials. The direct solutions in constant external field are Doppler-displaced plane waves propagating at the velocity of light c ; at the de Broglie wavelength scale and expressed in terms of the dynamically equivalent and appropriate geometric mean wave variables, these render as functions identical to the space-time functions of a corresponding Dirac spinor, and in turn to de Broglie phase waves previously obtained from explicit superposition. For two spin-half particles of a common set of space-time functions constrained with antisymmetric spin functions as follows the Pauli principle for same charges and as separately indirectly induced based on experiment for opposite charges, the complete wave functions are identical to the Dirac spinor. The back-substitution of the so explicitly determined complete wave functions in the corresponding classical wave equations of the two particles, subjected further to reductions appropriate for the stationary-state particle motion and to rotation invariance when in three dimensions, give a Dirac equation set; the procedure and conclusion are directly extendible to arbitrarily varying potentials by use of the Furioso theorem and to three dimensions by virtue of the characteristics of de Broglie particle motion. Through the derivation of the Dirac equation, the study hopes to lend insight into the connections between the Dirac wave functions and the electrodynamic components of simple particles under the government by the well established basic laws of electrodynamics.

1. Introduction

P.A.M. Dirac established in [1] a relativistic quantum mechanical wave equation, Dirac equation, for a point electron based on the relativistic energy-momentum relation subjected to Lorentz transformation under rotation. In [1] P.A.M. Dirac also theoretically predicted for the electron the existence of an internal oscillation state, a magnetic moment, and by interpretation of the negative energy solution, an anti-particle state known today as the positron. The Dirac equation has proven to be an accurate equation of motion for (two) spin-half quantum particles at high velocities; most notably, Dirac predicted based on his equation the relativistic intensities of Zeeman components of spectral lines and the frequency differences [1 (1928b)] in exact agreement with experiment. Up to the present however it has remained an open question that what is waving with the Dirac wave functions, or Dirac spinor, a similar question as for the Schrödinger wave functions and the de Broglie waves ? In addition, the Dirac theory meets with a few its own open questions. What is the nature of a Dirac internal oscillation? How are the Dirac space-time functions explicitly connected with the spin orientations, the signs of charges, the signs of the energies, and in the extreme situation when an electron and positron annihilate, the emitted two gamma rays and conversely? What is the symmetry of the total spin of an electron and positron? Also, in the case of an isolated single electron or positron in zero external field where the spin orientation is of no consequence, it would be desirable to have a way to directly write down the corresponding Dirac equation without involving the Pauli matrices. These as well as various other not fully addressed questions relating to fundamental physics seem to consistently point to the inadequacy of the point particle picture of today and the need for a representation of the internal processes of the particles.

Recently, using overall experimental observations as input data the author proposed an internally electrodynamic (IED) particle model [2a] (with coauthor P.-I. Johansson) or sometimes termed a basic particle formation (BPF) scheme, which states that *a simple (basic) particle like an electron and positron, etc., briefly, is constituted of an oscillatory point-like (elementary) charge with a specified sign and a zero rest mass, and the resulting electromagnetic waves in the vacuum.* As a broad test of the IED particle model and also as an endeavour of understanding the various puzzles relating to fundamental physics, in terms of solutions for the electrodynamic processes of the model particle with its charge's sign and total energy as two sole input data, the author has further achieved with coauthor(s) derivations/predictions of a range of basic properties and relations of the simple particles [2a-j] including the relativistic mass, de Broglie wave, de Broglie relations, Schrödinger equation, Einstein energy-mass relation, Newton's law of gravity and Doebner-Goldin equation, among others. As to the Schrödinger wave function specifically relevant here, the solution[2a,c] showed that it is the (envelope of the) standing wave, superposed from the Doppler-differentiated electromagnetic waves generated by the particle's travelling source charge, that is waving.

As previously shown e.g. in [2c], the direct solutions for the classical wave equations,

derivable from the Maxwell's equations, for the electromagnetic waves comprising a free particle consist of Doppler-displaced plane waves; these superpose to two opposite-travelling beat waves that resemble directly the de Broglie phase waves and in turn the Dirac space-time functions which in common are functions of the particle's total energy and linear momentum and thus are "relativistic". It therefore is foreseeable that the classical wave equations for the electromagnetic waves would more naturally lead to a wave equation of the particle corresponding directly to the Dirac equation in comparison to the Schrödinger equation. We elucidate in this paper a formal procedure which transforms the classical wave equations for the electromagnetic waves of two spin-half particles, of identical space-time functions and tending to approach one another, to the Dirac equation. Through the procedure we show that the Dirac internal oscillation corresponds to the oscillation of the electromagnetic waves at a geometric mean of frequencies which in general are Doppler-displaced owing to source motion, and we elucidate the explicit relationships between the internal electromagnetic waves, charges, spins, the centre-of-mass and total wave motions and the associated energies of the particles under the government of a few established elementary laws of electrodynamics.

2. Wave equations for the electromagnetic waves of particle. Solutions

We consider an IED particle, here an electron or positron, is as its source charge q ($= e$ or $-e$) travelling at a velocity \mathbf{v} in $+z$ -direction for the present along a one-dimensional box of side L in the vacuum. The charge q of the particle has an oscillation associated with a total energy ε_q , which is minimum at $v = 0$, denoted by \mathcal{E}_q ; \mathcal{E}_q may be endowed e.g. in a pair production in the vacuum. In virtue that it describes the ground state, \mathcal{E}_q cannot be dissipated or detached from the charge except in a pair annihilation.

The charge q of the particle generates owing to its oscillation electromagnetic waves of radiation electric fields \mathbf{E}^j 's and magnetic fields \mathbf{B}^j 's described in zero applied potential field by the Maxwell's equations as:

$$\nabla \cdot \mathbf{E}^j = \rho_q^j / \epsilon_0, \quad \nabla \cdot \mathbf{B}^j = 0, \quad \nabla \times \mathbf{B}^j = \mu_0 \mathbf{j}_q^j + (1/c^2) \partial_t \mathbf{E}^j, \quad \nabla \times \mathbf{E}^j = -\partial_t \mathbf{B}^j. \quad (41)$$

Where ρ_q^j is the density and \mathbf{j}_q^j the current of the particle's charge, assuming no other charges and currents present; ϵ_0 is the permittivity and μ_0 the permeability of the vacuum, and c is the velocity of light; $\partial_t \equiv \partial/\partial t$. Expressing the j th fields generally by a dimensionless displacement φ^j , $E^j = D\varphi^j$, thus $B^j = E^j/c = D\varphi^j/c$, with D a conversion constant, considering regions sufficiently away from the source only so that $\rho_q^j = j_q^j = 0$, and with some otherwise standard algebra of the Maxwell's equations (41), we obtain the corresponding classical wave equations for the electromagnetic waves φ^j 's

$$c^2 \nabla^2 \varphi^j = \partial_t^2 \varphi^j, \quad (42)$$

with $\partial_t^2 \equiv \partial^2/\partial t^2$. In the above, $j = \dagger$ labels the component wave generated in the direction parallel with $+v$, and $j = \ddagger$ the wave parallel with $-v$; within walls there

prevail also their reflected components generated by the reflected charge at an earlier time and being at the present time as if generated by a virtual charge travelling in the $-z$ -direction, labelled by $j = \text{vir}\dagger$ and $j = \text{vir}\ddagger$. j is to distinguish a Doppler effect owing to the source motion to be expressed in (47) below. In Appendix A we outline in relevance to the particle model a few further standard relations of classical and quantum electrodynamics for the electromagnetic waves, and a derivation of the particle's mass given by the author previously[2a,e] (with P.-I. Johansson).

To the particle we now apply an electromagnetic force $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m$, with $\mathbf{F}_e = -q\nabla\phi_a$ the Coulomb force in z -direction and $\mathbf{F}_m = -q\mathbf{v} \times \nabla \times \mathbf{A}_a$ the Lorentz force due to an external scalar potential ϕ_a and vector potential \mathbf{A}_a , expressed in SI units as for all other quantities in this paper. \mathbf{F}_m may be simplified using the BAC-CAB rule as $\mathbf{F}_m = -q[\nabla(\mathbf{v} \cdot \mathbf{A}_a) - \mathbf{A}_a(\mathbf{v} \cdot \nabla)]$. In the applications below \mathbf{A}_a is constant in L or in each small division in question (see end of Sec. 3), and v is constant in L for the particle being in stationary state and also is parallel with ∇ and \mathbf{z} , so $\nabla(\mathbf{v} \cdot \mathbf{A}_a) = 0$ and $\mathbf{F}_m = q\mathbf{A}_a(\mathbf{v} \cdot \nabla) = q\mathbf{A}_a v \nabla$. Thus, $\mathbf{F} = -q\nabla\phi_a + q\mathbf{A}_a v \nabla$. The formula of \mathbf{F} is in the usual usage established for a point particle; so when extending to the extensive IED particle here, \mathbf{F} apparently directly acts on the particle's point charge.

We need to map the \mathbf{F} to a force directly interacting with the internal fields E^j, B^j , or φ^j of the particle. We observe that, in virtue of its form, (42) represents just a classical wave equation for a mechanical wave of a transverse displacement $a\varphi^j$ propagated in an apparent *elastic medium*, a being a conversion factor of length dimension and apparently being cancelled in (42). On grounds of this direct correspondence, but taken as a heuristic means only in this paper (so that we here need not involve the details of this elastic medium), \mathbf{F} therefore interacts with the internal fields through a force $\mathbf{F}_{\text{med}}^j$ directly acting on this apparent medium. We can think of the medium to be composed of coupled dipole charges which do not move along the z -axis but the $\mathbf{F}_{\text{med}}^j$ propagates across the dipoles at the wave speed c . If viewing in a frame where $\mathbf{F}_{\text{med}}^j$ is at rest, then effectively the dipole charges are travelling at the speed c ; so as a first step of mapping, the Lorentz force on the medium ought to scale as $\mathbf{F}_{\text{med}}^{j'} = (\pm c/v)\mathbf{F}_m = \pm q\mathbf{A}_a c \nabla$ with $+, -$ for the $j = \dagger, \ddagger$ waves; thus $\mathbf{F}^{j'} = \mathbf{F}_e + \mathbf{F}_{\text{med}}^{j'}$. Under the actions of the respective forces, the acceleration $F^{j'}/m^j$ of the particle's charge of a dynamical mass m^j (due to the charge's total motion and equivalently the φ^j motion, see further Appendix A), and that of the medium of a dynamical mass \mathfrak{M}_{φ}^j , $F_{\text{med}}^j/\mathfrak{M}_{\varphi}^j$ must equal, i.e. $\mathbf{F}^{j'}/m^j = \mathbf{F}_{\text{med}}^j/\mathfrak{M}_{\varphi}^j$. Thus $\mathbf{F}_{\text{med}}^j = \frac{\mathfrak{M}_{\varphi}^j}{m^j} \mathbf{F}^{j'} = \frac{\mathfrak{M}_{\varphi}^j q}{m^j L_{\varphi}^j} (-\nabla\phi_a \pm \mathbf{A}_a c \nabla)$.

By its pure mechanical virtue the force $\mathbf{F}_{\text{med}}^j$ acting on the continuous medium is nonlocal and will be transmitted uniformly across the medium here along the z -axis of effective lengths $L_{\varphi}^{\dagger}, L_{\varphi}^{\ddagger}$ for the $j = \dagger, \ddagger$ waves (φ^j winds J^j loops about L). Using the geometric mean $L_{\varphi} = \sqrt{L_{\varphi}^{\dagger} L_{\varphi}^{\ddagger}}$, thus $\nabla\phi_a = \pm(\phi_a/L_{\varphi})\hat{z}$ and $\nabla = \pm 1/L_{\varphi}$. With these, putting $\mathfrak{M}_{\varphi}^j = \rho_i L_{\varphi}^j$ where ρ_i is the (geometric mean) linear mass density of the medium, writing for conciseness $\nabla\phi_a$ and also the final $\mathbf{F}_{\text{med}}^j$ in scalar forms and keeping

the generally arbitrarily oriented \mathbf{A}_a in vector form only, $\mathbf{F}_{\text{med}}^j$ becomes

$$F_{\text{med}}^j = -\rho_l V^j/m^j, \quad V^\dagger = q\phi_a - q\mathbf{A}_a c, \quad V^\ddagger = -q\phi_a - q\mathbf{A}_a c. \quad (43)$$

F_{med}^j can be implemented in (42) by directly establishing the corresponding wave equation for the apparent elastic medium acted by F_{med}^j . If without F_{med}^j , the elastic medium would be deformed owing to the disturbance of the oscillation of the source charge alone, by a total displacement $u = a \sum_j \varphi_{v'}^j$, and be thus subject to a tensile force $F_R = \rho_l c^2$. The applied F_{med}^j and F_R add up to a total force acting on the particle through acting directly on the medium

$$F_R^{j'} = F_R - F_{\text{med}}^j = \rho_l [c^2 + V^j/m^j]. \quad (44)$$

Where, the minus sign of F_{med}^j is because this force tends to contract the chain. Assuming φ^j is relatively small which in general is the case in practical applications, $F_R^{j'}$ is thus uniform across the L . A segment ΔL of the medium along the box, of mass $\Delta \mathfrak{M}_\varphi = \Delta \mathfrak{M}_\varphi^j/J^j = \rho_l \Delta L \simeq \rho_l \Delta z$, will upon deformation be tilted from its equilibrium position z -axis an angle ϑ^j and $\vartheta^j + \Delta \vartheta^j$ at z and $z + \Delta z$. The transverse (y -) component force acting on $\Delta \mathfrak{M}_\varphi$ is $\Delta F_{Rt}^{j'} = F_{Rt}^{j'} [\sin(\vartheta^j + \Delta \vartheta^j) - \sin \vartheta^j] = F_{Rt}^{j'} \nabla^2(a\varphi^j) \Delta z = a\rho_l [c^2 + \frac{V^j}{m^j}] \nabla^2 \varphi^j \Delta z$. Newton's second law for the mass $\Delta \mathfrak{M}_\varphi$ writes $\rho_l \Delta z \partial_t^2(a\varphi^j) = \Delta F_{Rt}^{j'}$. The two last equations give the equations of motion, on dividing $a\rho_l \Delta z$, for per unit length per unit linear mass density of the medium at z or equivalently the classical wave equations for the electromagnetic waves φ^j 's in the fields of the applied potentials ϕ_a, \mathbf{A}_a :

$$[c^2 + q(\phi_a - \mathbf{A}_a c)/m^\dagger] \nabla^2 \varphi^\dagger = \partial_t^2 \varphi^\dagger, \quad [c^2 - q(\phi_a + \mathbf{A}_a c)/m^\ddagger] \nabla^2 \varphi^\ddagger = \partial_t^2 \varphi^\ddagger. \quad (45)$$

This for $\phi_a = A_a = 0$ reduces to (42) given directly from the Maxwell's equations earlier.

Assuming for the present ϕ_a, \mathbf{A}_a are constant and also \mathbf{A}_a is small such that the particle motion effectively deviates not from the linear path, so the solution of (45) consists of plane waves $\varphi^\dagger = \mathcal{C}f^\dagger$ and $\varphi^\ddagger = \mathcal{C}f^\ddagger$ (Figure 1a, solid and dotted curves) generated in $+z$ - and $-z$ - directions and initially also travelling in these directions at speed $\omega^j/k^j = c$, with

$$f^\dagger = C e^{i[k_d^\dagger z - \omega^\dagger t + \alpha_0]}, \quad f^\ddagger = -C e^{i[-k_d^\ddagger z + \omega^\ddagger t - \alpha_0]} \quad (46)$$

(Figure 1 a-b, single-dot-dashed and triple-dot-dashed curves), $\mathcal{C} = e^{iKz}$, and $C (= 4C_1/\sqrt{L})$ a normalisation constant. Where,

$$k^\dagger = K/(1 - v/c) = \gamma^\dagger K, \quad k^\ddagger = K/(1 + v/c) = \gamma^\ddagger K \quad \text{and} \quad \omega^\dagger = \gamma^\dagger \Omega, \quad \omega^\ddagger = \gamma^\ddagger \Omega \quad (47)$$

are the source-motion resultant Doppler-displaced wavevectors and angular frequencies; $\gamma^\dagger = \frac{1}{1-v/c}$, $\gamma^\ddagger = \frac{1}{1+v/c}$; $K, \Omega = Kc$ are values of k^j, ω^j at $v = 0$. (47) further gives

$$k_d^\dagger = k^\dagger - K = \gamma^\dagger K_d, \quad k_d^\ddagger = K - k^\ddagger = \gamma^\ddagger K_d \quad \text{where} \quad K_d = (v/c) K. \quad (48)$$

Supposing $v \ll c$ (yet v^2/c^2 may be large so that dynamically the γ factor in (49) below can be different from 1) and accordingly the de Broglie wavelength ($\lambda_d = 2\pi/(\gamma K_d)$) later will be much greater than the electromagnetic wavelength ($\lambda = \frac{2\pi}{K}$), so at the scale of λ_d the rapid variation of \mathcal{C} is to an external observer no different from the constant 1, that is $\lim_{v \ll c} \mathcal{C} = 1$. Thus $\varphi^\dagger|_{\mathcal{C}=1} = f^\dagger$, $\varphi^\ddagger|_{\mathcal{C}=1} = f^\ddagger$ and the f^\dagger, f^\ddagger as given in (46) represent external-effective space-time functions. We see that f^\dagger and f^\ddagger are two new plane waves travelling each to the right at equal phase velocities, $\frac{\omega^\dagger}{k_d^\dagger} = \frac{-\omega^\ddagger}{-k_d^\ddagger} = W = \frac{c^2}{v}$ which is c/v times the velocity of light c . It can be checked (Appendix B) that the exact solutions φ^j 's and in turn the external effective f^j 's given above placed in the respective wave equations (45) above and (52) below yield exactly the expected relativistic energy-momentum relation.

The Doppler-displaced variables k_d^j, ω^j 's in the f^j 's, φ^j 's are not single valued and thus are not good dynamical variables of the particle. The respective geometric means

$$k_d = \sqrt{k_d^\dagger k_d^\ddagger} = \gamma K_d, \quad \omega = \sqrt{\omega^\dagger \omega^\ddagger} = \gamma \Omega, \quad \text{with } \gamma = \sqrt{\gamma^\dagger \gamma^\ddagger} = 1/\sqrt{1 - v^2/c^2}, \quad (49)$$

are evidently good dynamical variables of particle and also are appropriate in view of the stochastic virtue of the electromagnetic waves. These are also the natural independent variables of the superposed wave functions (for a detailed elucidation see [2c,d]): $\tilde{\psi} = \varphi^\dagger + \varphi^\ddagger = \mathcal{C}_d e^{i[k_d z - \omega t + \alpha_0]}$ and $\tilde{\psi}^{\text{vir}} = \varphi^{\text{vir}\dagger} + \varphi^{\text{vir}\ddagger} = -\mathcal{C}_d^{\text{vir}} e^{i[k_d z + \omega t + \alpha_0]}$ which are two beat waves of a wavelength $\lambda_d = 2\pi/k_d$, travelling each at the phase speed $W = c^2/v$ to the right and to the left as due to the actual and reflected (virtual) charges travelling to the right and left respectively; $\mathcal{C}_d = 2C_1 e^{i[(K + \frac{v}{c}K_d)z - \frac{v}{c}\omega t]}$, $\mathcal{C}_d^{\text{vir}} = 2C_1 e^{i[(K + \frac{v}{c}K_d)z + \frac{v}{c}\omega t]}$ and $\mathcal{C}_d \doteq \mathcal{C}_d^{\text{vir}} \doteq 2C_1 e^{iKz} = 2C_1$ for $v \ll c$. An inspection will show that clearly $\tilde{\varphi}, \tilde{\varphi}^{\text{vir}}$ resemble directly the de Broglie phase waves of the particle in the constant ϕ_a, \mathbf{A}_a fields here, λ_d and k_d are the de Broglie wavelength and wavevector, and accordingly $\hbar k_d = p_v$ the linear momentum. If ϕ_a, \mathbf{A}_a are arbitrarily varying in L and well behaved, we can divide L in a large N number of small divisions in each of which the plane waves remain true and their sum gives according to the Fourier theorem the total wave, and in three dimensions the de Broglie particle motion is a straightforward extension of a locally one-dimensional motion; the wave equations to be given will formally be otherwise the same (for a formal treatment in the case of a Schrödinger system see [2c,k]). We shall thus for simplicity proceed the remainder of treatment for constant ϕ_a, \mathbf{A}_a and, until the discussion regarding spin rotation in Sec. 5, for a particle motion in one-dimension.

3. Wave equation for total motion of particle

For single particle or for many particles without regarding the spins, the functions $\tilde{\psi}$ and $\tilde{\psi}^{\text{vir}}$, or equivalently the f_r and f_ℓ of (55) below, are seen to be identical to the usual solutions to the Dirac equation, c.f. Appendix C. So their wave equations, originally the (45), evidently must have a direct correspondence with the Dirac equation. The remainder of the task mainly will be to identify a physically justifiable procedure to transform (45) to a form of the Dirac equation under corresponding considerations.

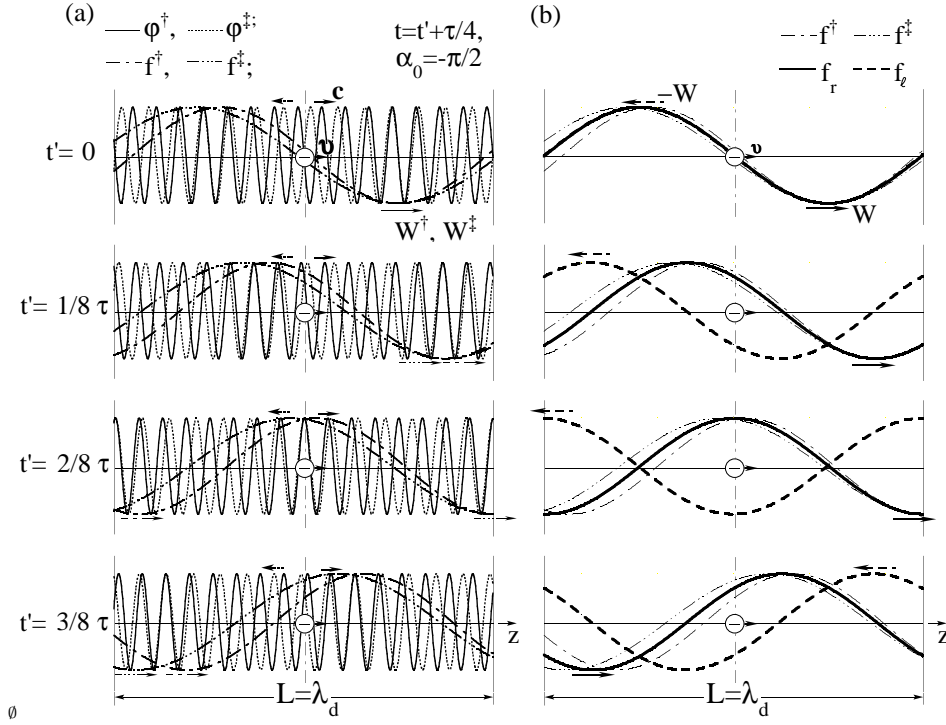


Figure 1. (a) shows an IED model electron constituted of an oscillatory charge $-e$ of zero rest mass (\ominus), travelling at velocity v , and the resulting Doppler-differentiated electromagnetic waves φ^\dagger and φ^\ddagger (solid and dotted curves) of a angular frequencies $\omega^\dagger, \omega^\ddagger$ generated in $+z$ - and $-z$ - directions, plotted for the real parts in a time interval $(3/4)\tau$ in a one-dimensional box of side $L = \lambda_d$; $\tau = 2\pi/\omega$, $\omega = \sqrt{\omega^\dagger \omega^\ddagger}$, $\lambda_d = 2\pi((v/c)\omega)$ being the de Broglie wavelength. f^\dagger and f^\ddagger (single-dot-dashed and triple-dot-dashed curves) are the corresponding external-effective waves, shown in both (a) and (b). f_r and f_l (solid and dashed curves) in (b) are the dynamically equivalent mean-variable wave functions; these resemble directly the (opposite-travelling) de Broglie phase waves and are equivalent to the space-time functions of Dirac spinor.

First, similarly as Dirac (or as alternatively but compatibly argued in [2c]) we want the eventual wave functions of the particle, and thus immediately the f^j or the original φ^j to be linear in $\hbar\omega$ and thus $\partial_t f^j$ here, so that f^j at any initial time determines its value at any future time; and we want similarly for the linear momentum here. We shall thus transform the second order differential equations (45) to first order ones and in the end take the limit for $c \gg v$ as follows. For the $c^2 \nabla^2 \varphi^j$ terms of (45), starting with the full wave functions $\varphi^j = \mathcal{C}f^j$ we first lower the spatial derivative one order as $\nabla^2 \varphi^j = \nabla[i\gamma^j K \mathcal{C}f^j] = i\gamma^j K[(\nabla \mathcal{C})f^j + \mathcal{C}\nabla f^j]$. Restricting to $v \ll c$, we thus can replace $\nabla \mathcal{C}$ by its computed value $iK\mathcal{C}$ and in turn put $\mathcal{C} \doteq 1$ for each term; this gives

$$\nabla^2 \varphi^j|_{\mathcal{C} \doteq 1} = [-\gamma^j K^2 \mathcal{C}f^j + i\gamma^j K \mathcal{C}\nabla f^j]|_{\mathcal{C} \doteq 1} = -\gamma^j K^2 f^j + i\gamma^j K \nabla f^j, \quad j = \dagger, \ddagger. \quad (50)$$

Next, assuming ϕ_a, \mathbf{A}_a relatively small as typically is true in applications, so the resulting force constant (i.e. force per unit displacement) on the particle does not vary across L ; we can thus replace the $\nabla^2 \varphi^j$ in the ϕ_a, \mathbf{A}_a terms of (45) by its computed value as

$\nabla^2 \varphi^j|_{\mathbb{C}=1} = -\gamma^{j2} K^2 f^j$, thus

$$\frac{q(\phi_a - \mathbf{A}_a c)}{m^\dagger} \nabla^2 \varphi^\dagger|_{\mathbb{C}=1} = q(-\phi_a + \mathbf{A}_a c) \frac{\gamma^\dagger \Omega f^\dagger}{\hbar}, \quad \frac{-q(\phi_a + \mathbf{A}_a c)}{m^\ddagger} \nabla^2 \varphi^\ddagger|_{\mathbb{C}=1} = q(\phi_a + \mathbf{A}_a c) \frac{\gamma^\ddagger \Omega f^\ddagger}{\hbar} \quad (51)$$

For the final expression we used $Kc = \Omega$ as earlier, and $m^j = \gamma^j M$ and $Mc^2 = \hbar\Omega$ given after (A.1) and (49). Finally, the $\partial_t^2 \varphi^j$'s of (45) lower one order as $\partial_t^2 \varphi^\dagger|_{\mathbb{C}=1} = -i\gamma^\dagger \Omega \partial_t f^\dagger$, $\partial_t^2 \varphi^\ddagger|_{\mathbb{C}=1} = i\gamma^\ddagger \Omega \partial_t f^\ddagger$. Substituting these and equations (50)–(51) in wave equations (45), multiplying the first resulting equation by $-\frac{\hbar c}{K\gamma^\dagger}$ and the second by $\frac{\hbar c}{K\gamma^\ddagger}$, with $cK = \Omega$ and $\hbar K = Mc$ as before and after (A.1), we eventually obtain the wave equations for the electromagnetic waves of the particle expressed by f^\dagger, f^\ddagger :

$$[Mc^2 + q\phi_a - c(i\hbar\nabla + q\mathbf{A}_a)]f^\dagger = i\hbar\partial_t f^\dagger, \quad [-Mc^2 + q\phi_a + c(i\hbar\nabla + q\mathbf{A}_a)]f^\ddagger = i\hbar\partial_t f^\ddagger. \quad (52)$$

For the particle dynamics in question we want to further transform (52) to be expressed by the particle wave variables, i.e. the k_d and ω defined in (49), and the corresponding wave functions, the f_r, f_ℓ to obtain below. We shall below obtain such functions through a dynamic equivalence transformation directly from the f^\dagger, f^\ddagger ; these ought to be and will show to be functions identical to the $\tilde{\psi}, \tilde{\psi}^{\text{vir}}$ obtained in a physically more transparent way earlier; the present approach below will advantageously preserve a direct tractable connection with the original f^\dagger, f^\ddagger , thus also $\varphi^\dagger, \varphi^\ddagger$, whose wave equations (52) or (45) give the relativistic energy-momentum relation exactly based on the Doppler equations (47), see (B.2) of Appendix B. What (accordingly) is in question in the transformation mainly is to maintain an equivalence to the quadratic equation (B.2); this corresponds to the equations $\frac{\partial f^j}{\partial z^\nu} \frac{\partial f^{j'}}{\partial z^\nu} = \frac{\partial f_\mu}{\partial z^\nu} \frac{\partial f_{\mu'}}{\partial z^\nu}$, etc., with $j, j' = \dagger, \ddagger$, $\mu, \mu' = r, l$, $z^\nu = t, z$ ($\nu = 0, 3$); and $f_\mu \frac{\partial f_{\mu'}}{\partial z} = 0$. The equivalence condition requires in particular the transformed quadratic to be $\frac{\partial f_r}{\partial z} \frac{\partial f_l}{\partial z} = k_d^2$, that is, it has a plus sign in front and its cross-term product with Mc^2 (i.e. the \mathcal{O} discussed after 57) is absent. This can be achieved if we introduce a wavevector being the imaginary of (thus orthogonal to) k_d :

$$\bar{k}_d = (\gamma/i\gamma^\dagger)k_d^\dagger, \quad \bar{k}_d = (\gamma/i\gamma^\ddagger)k_d^\ddagger; \quad \text{thus } k_d^\dagger k_d^\ddagger = (1/i^2)\bar{k}_d^2 \quad (53)$$

(compare \bar{k}_d with the operator $p_{v.op} = \frac{\hbar}{i}\nabla$ later). (53) alternatively can be expressed by

$$(a) : k_d^\dagger(-k_d^\ddagger) = \bar{k}_d \bar{k}_d \quad \text{or} \quad (b) : (-k_d^\ddagger)k_d^\dagger = \bar{k}_d \bar{k}_d. \quad (54)$$

We now first transform the Doppler-differentiated $f^j(; k_d^j, \omega^j)$'s (as short hand notations of $f^j(z, t; k_d^j, \omega^j)$'s) to a pair of mean (wave)-variable functions $f_\mu(; \bar{k}_d, \omega)$'s (denoting $f_\mu(z, t; \bar{k}_d, \omega)$'s) by, say, satisfying (a) of (54) and ordinarily $\omega = \sqrt{\omega^\dagger \omega^\ddagger}$ of (49):

$$f^\dagger(; k_d^\dagger, \omega^\dagger) \rightarrow f_r(; \bar{k}_d, \omega) = C_r e^{i[\bar{k}_d z - \omega t + \alpha_0]}, \quad f^\ddagger(; k_d^\ddagger, \omega^\ddagger) \rightarrow f_\ell(; \bar{k}_d, \omega) = C_\ell e^{i[\bar{k}_d z + \omega t + \alpha_0]}; \quad (55)$$

see these functions plotted in Figure 1b. The transformed f_r, f_ℓ indeed are desirably identical functions to the original f^\dagger, f^\ddagger if disregarding the high-order differences in the coefficients $\gamma^\dagger, \gamma^\ddagger$ and γ in the wave variables and the reversed travel direction

of f_ℓ from f^\dagger . The f_r, f_ℓ , being identical functions to the $\tilde{\psi}, \tilde{\psi}^{\text{vir}}$ earlier, indeed are therefore the pertinent space-time functions of the particle; these are each functions of the source motion and the total (electromagnetic) wave oscillation and accordingly directly resemble the de Broglie phase waves; and these are equivalent to Dirac's space-time functions.

To entail that in the matrix representation later (Sec. 5) a cross-term product \mathcal{O} discussed after (57) is similarly absent, for transformation of the first derivatives we are compelled to satisfy the alternative condition (b) of (54). The use of (54b) and ordinarily the $\omega = \sqrt{\omega^\dagger \omega^\dagger}$ of (49) first directly leads to the intermediate transformations for the f^j 's given in the left column below:

$$\begin{aligned} f^\dagger &\xrightarrow[k_d^\dagger \rightarrow -\bar{k}_d, \alpha_0 \rightarrow -\alpha'_0]{} f_\ell^* \xrightarrow{(a1)} f_\ell, & \nabla f^\dagger &= i k_d^\dagger f^\dagger \xrightarrow[k_d^\dagger \rightarrow -\bar{k}_d, f^\dagger \rightarrow f_\ell]{} i(-\bar{k}_d) f_\ell = -\nabla f_\ell, \\ f^\dagger &\xrightarrow[k_d^\dagger \rightarrow \bar{k}_d, \alpha_0 \rightarrow \alpha'_0]{} -f_r^* \xrightarrow{(a2)} -f_r, & \nabla f^\dagger &= -i k_d^\dagger f^\dagger \xrightarrow[k_d^\dagger \rightarrow \bar{k}_d, f^\dagger \rightarrow -f_r]{} -i \bar{k}_d (-f_r) = \nabla f_r \end{aligned} \quad (56)$$

where, $f_\ell^* = C_\ell e^{i[-\bar{k}_d z - \omega t - \alpha'_0]}$, $f_r^* = C_r e^{i[-\bar{k}_d z + \omega t + \alpha'_0]}$, $\alpha'_0 = -\alpha_0$; the transformations (a1) and (a2) in (56) finally naturally lead to the same f_ℓ, f_r as in (55), which indeed also represent the original f_ℓ^*, f_r^* in all aspects (having the same phase velocities and wave forms) except the opposite rotating phases on the complex plane and the opposite signs of α'_0 and α_0 that altogether are dynamically inconsequential. The relations in the left column and the use of (54b) again then lead to the results in the right column which is actually in question in respect to dynamical equivalence here.

Substituting in wave equations (52) the transformation relations (55) and (56) gives

$$(Mc^2 + q\phi_a)f_r + c(i\hbar\nabla - q\mathbf{A}_a)f_\ell = i\hbar\partial_t f_r, \quad (-Mc^2 + q\phi_a)f_\ell + c(i\hbar\nabla + q\mathbf{A}_a)f_r = i\hbar\partial_t f_\ell. \quad (57)$$

As a check, placing in (57) the f_r, f_ℓ of (55), multiplying the first and the negative of the second resulting equations, dividing $f_r f_\ell$, putting for simplicity $\phi_a = A_a = 0$ for the problem mainly of concern here, we correctly obtain the same result as (B.2): $M^2 c^4 - \hbar^2 \bar{k}_d^2 c^2 + \mathcal{O} = \hbar^2 \omega^2$ where $\bar{k}_d^2 = -k_d^2$ following (53) and (49); $\mathcal{O} = 0$.

4. Two-particle system: spins, charges, and time-arrows

Consider two spin-half particles 1,2 having identical sets of $\{f_r, f_\ell\}$'s tend to occupy the same location z or more precisely the same region in $(0, L)$; suppose these are noninteracting (a finite particle-particle interaction can in principle be included in V^j and will not affect the general conclusions below). In virtue of the statistical nature of the electromagnetic displacements, the probability of finding a portion of particles 1 and 2 at locations z_1 and z_2 is proportional to the product $f_\mu(z_1, t)f_{\mu'}(z_2, t)$. Since these have identical space-time function sets, the corresponding total space-time function is evidently symmetric, thus f_s , here in the only form $f_s(z_1, z_2) = \frac{1}{\sqrt{2}}[f_r(z_1, t)f_\ell(z_2, t) + f_\ell(z_2, t)f_r(z_1, t)]$ to be compatible with the antisymmetric total spin function later.

In the case of two identical electrons (Figure 2, panel a), their spins then need according to Pauli principle be opposite (left graph in the figure) to avoid both particles occupying the same quantum state; the total spin function for this is antisymmetric ($\chi_a = \frac{1}{\sqrt{2}}[\alpha(1)\beta(2) - \alpha'(2)\beta'(1)]$). Apparently it is in general also relevant that we introduce charge functions $\mathcal{Q}_-(1), \mathcal{Q}_-(2)$ to reflect the signs of charges 1,2; these for the two like charges are trivial identities and lead to a trivial symmetric total charge function, \mathcal{Q}_s . The above functions together define an antisymmetric two-electron function (Figure 2a, right graph), $\psi_a = f_s \mathcal{Q}_s \chi_a$, yielding as expected a total probability independent of how we sample the two stationary-state identical, indistinguishable (as result of being identically extensively distributed in L) particles.

In the case of an electron and positron (Figure 2, panel b), the total two-particle function is symmetric (right graph), thus ψ_s , as follows from the observational fact that two such particles can approach each other arbitrarily close and, in an extreme case

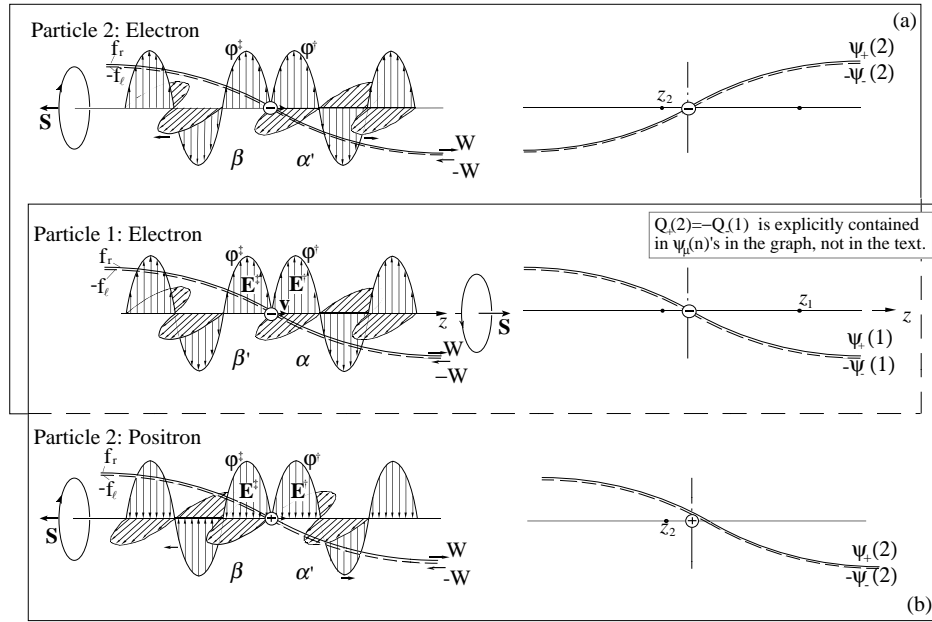


Figure 2. Two spin-half IED particles 1 and 2 described by identical, thus mutually symmetric sets of Doppler-displaced space-time functions $\{\varphi^\dagger, \varphi^\ddagger\}$'s or effectively $\{f_r, f_\ell\}$'s tend to occupy the same location z ; panel (a) shows two electrons and (b) an electron and positron. Particle 1 has a spin \mathbf{S} in $+z$ -direction, denoted spin up, $\alpha(1)$, and is parallel with the generating direction of the f_r wave; particle 2 has spin \mathbf{S} in $-z$ -direction, denoted spin down $\beta(2)$, and is parallel with f_ℓ ; their opposite counterparts (termed virtual spins) $\beta'(1)$ and $\alpha'(2)$ are parallel with f_ℓ and f_r . Right graphs show the complete wave functions $\{\psi_+(1), \psi_-(1)\}$ and $\{\psi_+(2), \psi_-(2)\}$ of particles 1 and 2. In panel (a), $\psi_\nu(1)$ and $\psi_\nu(2)$ ($\nu = +, -$) are antisymmetric due to the antisymmetric spin functions and same charges. In (b), these are symmetric (as shown in the graph) due to antisymmetric spins and also opposite charges ($\mathcal{Q}_+(2) = -\mathcal{Q}_-(1)$), the latter of which leads to the radiation electric fields \mathbf{E}^j 's are opposite in direction (in the text the $\mathcal{Q}_+(2), \mathcal{Q}_-(1)$ are not explicitly regarded, rendering the $\psi_\nu(1)$ and $\psi_\nu(2)$ for the electron and positron to be antisymmetric and the same as for two electrons).

annihilate into "one point" in the vacuum. The total charge function for their two opposite signed charges evidently needs be antisymmetric, thus Q_a . (If imagine the wave displacement φ^j in the medium is executed by a chain of dipole charges then it is immediately clear that the corresponding radiation electric field E^j , with $|E^j| \propto \varphi^j$, produced by the positive charge is reversed from that by the negative charge, see left graph in Figure 2b.) Placing the two known functions in $\psi_s = Q_a f_s \chi_{\text{Sym}}$ gives therefore Sym=antisymmetric and thus $\chi_{\text{Sym}} = \chi_a$ for the electron-positron system.

Our particles contain each the electromagnetic waves $\varphi^\dagger, \varphi^\ddagger$, or effectively f_r, f_ℓ , generated in the specified $+z$ - and $-z$ - directions which assume definite relationships with the spin orientations that turn out in a measurement: Given, say, particle 1 is measured to be spin up, $\alpha(1)$, with f_r being parallel with it, then its other internal process, f_ℓ , is parallel with a virtual (indicated by a prime) spin-down state, $\beta'(1)$. Similarly for particle 2, being then actually spin down, thus f_ℓ parallel with $\beta(2)$, its f_r is parallel with a virtual spin-up state, $\alpha'(2)$. From the foregoing antisymmetric spin requirement follow the relations for the spin functions:

$$\alpha'(2) = -\alpha(1), \quad \beta'(1) = -\beta(2); \quad \text{with} \quad \beta'(1) = -\alpha(1), \quad \alpha'(2) = -\beta(2) \quad (58)$$

following the opposite signs as is meant by the "virtual" spins. Through (58), the virtual spin vector of particle 1, $\mathbf{S}'(1)$ (virtual spin down) and the actual spin vector of particle 2, $\mathbf{S}(2)$ (actual spin down), pointed each in the $-z$ - direction, are now each represented as scalar quantities with minus signs.

If disregarding the signs of charges explicitly, the electron-electron and the electron-positron are two equivalent systems of identical, spin-half particles, each described by the space-time functions $f_\mu(; \bar{k}_d, \omega)$'s with $\mu = r, \ell$, which joined together with the spin functions give the complete wave functions: $\psi_\nu(n; \bar{k}_d, \omega) = f_\mu(; \bar{k}_d, \omega) \alpha_\nu(n)$ with $\nu = +, -$, $n = 1, 2$, $\alpha_\nu = \alpha, \beta$. Now as a further step to conform our wave equation later to matrix form, we hereafter require that the $\psi_\nu(n)$ functions are elements of a matrix of one column, $\boldsymbol{\psi}$. The matrix wave equation itself will entail the two desired features discussed after equation (57), that is, (i) the product of $-k_d$ and k_d in the quadratic equation is positive: k_d^2 (entailed by the situation that these in the matrix form are offdiagonal elements, see (62) or (C.1), and (ii) the total cross-term product $\mathcal{O} = 0$ (entailed by the characteristics that in matrix equation the $\psi_\nu(n)$'s are explicitly mutually orthogonal). The first of these two features which we have up to now enforced by use of \bar{k}_d for k_d , should no longer be used in the matrix form to avoid a dual accounting. The space-time functions accordingly write $f_r(; k_d, \omega) = C_r e^{i[k_d z - \omega t]}$, $f_\ell(; k_d, \omega) = C_\ell e^{i[k_d z + \omega t]}$ with k_d the ordinary scalar quantity and related with $k_d^\dagger, k_d^\ddagger$ through (49). Accordingly,

$$\psi_+(1) = \alpha(1) f_r(; k_d, \omega) = \alpha(1) C_r e^{i[k_d z - \omega t]}, \quad \psi_-(2) = \beta(2) f_r(; k_d, \omega) = \beta(2) C_r e^{i[k_d z - \omega t]}, \quad (59a)$$

$$\psi_-(1) = \beta'(1) f_\ell(; k_d, \omega) = \beta'(1) C_\ell e^{i[k_d z + \omega t]}; \quad \psi_+(2) = \alpha'(2) f_\ell(; k_d, \omega) = \alpha'(2) C_\ell e^{i[k_d z + \omega t]}. \quad (59b)$$

The complete wave functions (59a)–(b) (Figure 2, right graphs) describe two identical particles of opposite oriented actual spins and accordingly opposite virtual spins, and

these are identical to the solutions (see equation C.3) for Dirac equation. We can readily check that placing the foregoing relations in the antisymmetric total function for two spin-half (like charge) particles, $\psi_a = \frac{1}{\sqrt{2}}[\psi_+(1)\psi_-(2) - \psi_-(1)\psi_+(2)]$ correctly leads to that the probability of finding two identical particles at any location z in L is not altered by interchanging the locations of the particles (the indistinguishability). We can also check that the same ψ_a is given by the product of the separate total functions: $\psi_a(z_1, z_2) = f_s(z_1, z_2)\chi_a(1, 2)$; notice that once we specified say particle 1 is spin up and 2 spin down, then $f_r(z_1, t)f_\ell(z_2, t)\beta'(1)\alpha'(2)$ and $f_\ell(z_1, t)f_r(z_2, t)\alpha(1)\beta(2)$ are zero since these do not describe the present reality.

Lastly, the spin-up state of particle 1, $\alpha(1)$, is associated with an effective electromagnetic wave f_r travelling to the right, thus $\partial_t f_r / f_r = -i\omega$, while the spin-up state of particle 2 with f_ℓ travelling to the left, thus $\partial_t f_\ell / f_\ell = i\omega$; the latter has as if a reversed time arrow relative to the former. We may introduce the time arrow functions defined for particles 1 and 2 as $\mathcal{T}(1) = 1, \mathcal{T}(2) = -1$, such that the action of these on the time derivatives project the wave propagations to be both in the $+z$ -direction: $\mathcal{T}(1)\partial_t \psi_\nu(1) = \partial_t \psi_\nu(1)$, $\mathcal{T}(2)\partial_t \psi_\nu(2) = -\partial_t \psi_\nu(2)$.

5. Dirac equation

For two identical, spin-half particles of identical sets of space-time functions f_r, f_ℓ described by wave equations (57) tending to occupy the same location z , we shall now express the corresponding wave equations in terms of the complete wave functions of Sec. 4. For particle 1, we thus multiply the first equation of (57) by $\alpha(1)$ and the second by $\beta'(1)$, act $\mathcal{T}(1)$ in front of the time derivatives, denote its charge by q_1 , and get

$$\begin{aligned} (Mc^2 + q_1\phi_a)f_r\alpha(1) + c(i\hbar\nabla - q_1\mathbf{A}_a)f_\ell(-\beta'(1)) &= i\hbar\mathcal{T}(1)\partial_t f_r\alpha(1), \\ (-Mc^2 + q_1\phi_a)f_\ell\beta'(1) + c(i\hbar\nabla + q_1\mathbf{A}_a)f_r(-\alpha(1)) &= i\hbar\mathcal{T}(1)\partial_t(f_\ell\beta'(1)). \end{aligned} \quad (60)$$

For particle 2, instead we multiply the first equation of (57) by $-\beta(2)$ and the second by $-\alpha'(2)$, act both equations by $\mathcal{T}(2)$, denote its charge by q_2 , and get

$$\begin{aligned} (Mc^2 + q_2\phi_a)f_r(-\beta(2)) + c(i\hbar\nabla - q_2\mathbf{A}_a)f_\ell(+\alpha'(2)) &= i\hbar\mathcal{T}(2)\partial_t f_r(-\beta(2)), \\ (-Mc^2 + q_2\phi_a)f_\ell(-\alpha'(2)) + c(i\hbar\nabla + q_2\mathbf{A}_a)f_r(+\beta(2)) &= i\hbar\mathcal{T}(2)\partial_t f_\ell(-\alpha'(2)). \end{aligned} \quad (61)$$

In the second terms in equations (60)–(61) we made the replacements $\alpha(1) \rightarrow -\beta'(1)$ and $\beta'(1) \rightarrow -\alpha(1)$, $\beta(2) \rightarrow -\alpha'(2)$, $\alpha'(2) \rightarrow -\beta(2)$ based on (58), to conform to the transformed space-time functions earlier.

Substituting in (60)–(61) with (59a)–(b) for the $\psi_\nu(n)$'s and the $\mathcal{T}(n)$'s expressed earlier, and, to form a direct contrast between the actual spin directions of the two particles, re-arranging the resulting four equations in the order of spin-up states of particles 1 and 2 first and then spin-down states of particles 1 and 2, we finally obtain a set of four coupled linear first order partial differential equations governing the motions

of the two particles in terms of $\psi_\nu(n)$:

$$\begin{aligned}
(Mc^2 + q_1\phi_a)\psi_+(1) - c(i\hbar\nabla - q_1\mathbf{A}_a)\psi_-(1) &= i\hbar\partial_t\psi_+(1) \quad (\text{particle 1, spin up}) \\
(Mc^2 - q_2\phi_a)\psi_+(2) + c(i\hbar\nabla + q_2\mathbf{A}_a)\psi_-(2) &= i\hbar\partial_t\psi_+(2) \quad (\text{particle 2, spin up}) \\
(-Mc^2 + q_1\phi_a)\psi_-(1) - c(i\hbar\nabla + q_1\mathbf{A}_a)\psi_+(1) &= i\hbar\partial_t\psi_-(1) \quad (\text{particle 1, spin down}) \\
(-Mc^2 - q_2\phi_a)\psi_-(2) + c(i\hbar\nabla - q_2\mathbf{A}_a)\psi_+(2) &= i\hbar\partial_t\psi_-(2) \quad (\text{particle 2, spin down}) \quad (62)
\end{aligned}$$

From the discussion of Sec. 4 that the $\psi_\nu(n)$'s and accordingly also their first derivatives are mutually orthogonal, it follows that the linear equations (62) are equivalent to a matrix equation. Supposing specifically the two particles are a positron and an electron and therefore $q_1 = q$, $q_2 = -q$, the matrix form of (62) is thus

$$H_{op}\boldsymbol{\psi} = i\hbar\partial_t\boldsymbol{\psi}, \quad \text{with } H_{op} = \mathbf{b}Mc^2 + q\phi_a + c\boldsymbol{\alpha}(\mathbf{p}_{v.op} - q\mathbf{A}_a) \text{ and } \mathbf{p}_{v.op} = -i\hbar\nabla \quad (63)$$

being the relativistic total Hamiltonian and linear momentum operators. Where,

$$\begin{aligned}
\mathbf{b} &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma}_z \\ \boldsymbol{\sigma}_z & 0 \end{pmatrix}, \quad \boldsymbol{\psi} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad \text{and} \\
I &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \boldsymbol{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \boldsymbol{\psi}_+ = \begin{pmatrix} \psi_+(1) \\ \psi_+(2) \end{pmatrix}; \quad \boldsymbol{\psi}_- = \begin{pmatrix} \psi_-(1) \\ \psi_-(2) \end{pmatrix}. \quad (64)
\end{aligned}$$

The off-diagonal elements of the matrix $\boldsymbol{\sigma}_z$, $\sigma_{z11}(=1)$ and $\sigma_{z22}(=-1)$ here correspond to the $\alpha(1) = 1$ and $\alpha'(2) = -1$ earlier. We see that, $\boldsymbol{\psi}$ is equivalent to a Dirac spinor, σ_z the z -component of Pauli matrices, and as a whole, equation (63) is identical to the Dirac equation for an electron-positron system equivalent to here. For the present case rotation transformation is trivial, so $\vec{\sigma} = \sigma_z\hat{x}$.

Suppose more generally the two particles' spin angular momenta, \mathbf{S} ($=\frac{\hbar}{2}\boldsymbol{\sigma}$)'s, are along an axis \mathbf{n} executing in general a precession about the z -axis at a fixed angle ($\arccos(\frac{S_z}{S})$). For each particle being in stationary state, its \mathbf{S} (similarly its magnetic moment $\boldsymbol{\mu}_s(=-e\mathbf{S}/m)$) as a vector quantity when in small rotations about the z -axis must maintain invariant with respect to its projection on the z -axis, $\mathbf{n} \cdot \mathbf{S} = \pm\frac{1}{2}\hbar$, and is Hermitian. In addition to the antisymmetric condition given by the σ_z of (64) above, an infinitesimal rotation transformation as such needs be unitary. A specific set of transformation matrices having these properties are known to be the Pauli matrices, σ_x, σ_y and σ_z of the standard expressions and σ_z as expressed in (64), $\boldsymbol{\sigma} = \sigma_x\hat{x} + \sigma_y\hat{y} + \sigma_z\hat{z}$. And, the unitary matrix $I\hat{z}$ about the z -axis naturally extends to a unitary matrix about the new \mathbf{n} -axis in three dimensions, given by $\vec{I} = I\hat{x} + I\hat{y} + I\hat{z}$. Substituting in (63) with $\boldsymbol{\sigma}$ and I for $\boldsymbol{\sigma}_z$ and I gives a Dirac equation of the same form, now for spins in arbitrary directions.

Appendixes IA-IC:

Appendix A. Total energy and inertia of particle wave

As a general result of classical electrodynamics based on solution to the Maxwell's equations combined with Lorentz force law, an electromagnetic wave j transmits at the speed of light c a wave energy ε^j and a linear momentum $p^j = \varepsilon^j/c$. Here, the amplitudes of ε^j , accordingly of p^j , E^j , B^j and φ^j , etc., are continuous values. Following M. Planck's discovery of quantum theory in 1901, it has been additionally understood that these quantities are *by nature* quantized in amplitudes; an electromagnetic wave of frequency $\omega/2\pi$ has an energy $\varepsilon = n\hbar\omega$, consisting in general of n momentum-space quanta, or photons, each of an energy $\hbar\omega$; and the classical continuous amplitude solutions to these are only approximations when n is large. In the present problem, in conformity with experiments, especially the pair processes, the electromagnetic wave comprising our basic particle has a "single energy quantum", $n = 1$; so $\varepsilon = \hbar\omega$. It has been further proven especially through quantum electrodynamics that the Maxwell's equations, and the subsequent classical wave equation (42) or (45), continue to hold, and the quantisation of the fields and wave energy etc. is the result of subjecting the canonical displacement and momentum, the $u(=a\varphi)$ and \dot{u} here, to the quantum commutation relation $[u, \dot{u}] = i\hbar$.

The total wave of our particle of a single "quantum energy level" $\hbar\omega$ in a one-dimensional box has, following the solution to the Maxwell's equations earlier [see after (42)], two components, φ^\dagger and φ^\ddagger , with their frequencies being Doppler-displaced to ω^\dagger and ω^\ddagger as a result of the source motion as given in (47), which are related to ω through (49). For the total wave comprising the particle, ε represents therefore a dynamical variable of the particle, here the total energy of the particle.

The electromagnetic waves, E^j, B^j 's or φ^j 's, rapidly oscillating at frequencies $(\omega^j/2\pi)$'s, of a geometric mean frequency $\omega/2\pi$ and wavelength $\lambda = c/(\omega/2\pi)$ will, when ignoring the detailed oscillation as will effectively manifest at some distance, appear as if being two rigid objects, wavetrains, travelling at the speed of light c . In view that their speed of travel, c , is *finite* as contrasted to infinite, the wavetrains have inevitably each *finite* inertial masses, m^j 's, thus an inertial mass $m = \sqrt{m^\dagger m^\ddagger}$ for the total wavetrain and hence its resulting particle. This mechanical depiction of the total wave, as a rigid "wavetrain", permits us at once to express according to Newtonian mechanics the linear momentum of the wavetrain to be $p = mc$. Combining this with the classical electrodynamic result $\varepsilon = pc$ above gives the kinetic energy of the wavetrain $\varepsilon = mc^2$, being equivalent to the Einstein's mass-energy relation. This energy and the Planck energy earlier ought to equal, thus

$$m = \hbar\omega/c^2; \quad \text{or at } v = 0: M = \hbar\Omega/c^2 \quad (\text{A.1})$$

with M the rest mass of the particle; combining (A.1) with (49) gives $m = \gamma M$. Combining (A.1) with $p = mc$ further gives $mc = (\hbar\omega/c^2)c = \hbar k$ and accordingly $Mc = \hbar K$, with $\omega = kc$, $\Omega = Kc$ and $k = \gamma K$ as earlier.

Appendix B. Relativistic energy-momentum relation for the electromagnetic waves of particle

Consider first the simpler case of $A_a = 0$. Placing in wave equations (52) with f^\dagger, f^\ddagger of (46), dividing the resulting first and second equations by f^\dagger and $-f^\ddagger$ and sorting give $Mc^2 + \hbar k_d^\dagger c = \hbar\omega^\dagger - q\phi_a$, $Mc^2 - \hbar k_d^\ddagger c = \hbar\omega^\ddagger + q\phi_a$. Multiplying gives

$$M^2 c^4 - \hbar^2 k_d^\dagger k_d^\ddagger c^2 + Q = \hbar^2 \omega^\dagger \omega^\ddagger - q^2 \phi_a^2 \quad (B.1)$$

Where $k_d^\dagger k_d^\ddagger = k_d^2$ and $\omega^\dagger \omega^\ddagger = \omega^2$ following (49); $Q = Mc^2 c \hbar (k_d^\dagger - k_d^\ddagger)$, with $k_d^\dagger - k_d^\ddagger = 2k_d(\frac{v}{c})\gamma$ and $Mc^2 = \hbar Kc$, so $Q = 2\hbar^2 k_d^2 c^2$. With these, putting $\hbar k_d = \pm p_v$, $\hbar\omega = \pm \varepsilon$ where p_v, ε are here variables having positive and negative solutions and thus the right hand side of (B.1) reduces as $\sqrt{[(\hbar\omega - q\phi_a)(\hbar\omega + q\phi_a)]^2} = \sqrt{[-(\varepsilon - q\phi_a)(\varepsilon + q\phi_a)]^2} = (\varepsilon - q\phi_a)^2$, then (B.1) reduces exactly to $M^2 c^4 + p_v^2 c^2 = (\varepsilon - q\phi_a)^2$. This, or this in the more familiar form for $\phi_a = 0$,

$$M^2 c^4 + c^2 p_v^2 = \varepsilon^2, \quad (B.2)$$

gives just the experimentally widely corroborated relativistic energy-momentum relation. For the more general case of \mathbf{A}_a finite, denoting $k_d^{\dagger'} = \bar{k}_d^\dagger - \frac{q\mathbf{A}_a}{\hbar}$, $k_d^{\ddagger'} = k_d^\ddagger + \frac{q\mathbf{A}_a}{\hbar}$, the particular feature that (the effective portion of) \mathbf{A}_a is always perpendicular to $k_d \mathbf{z}^0$ leads to $k_d'^2 = k_d^{\dagger'} k_d^{\ddagger'} = k_d^2 - q^2 \mathbf{A}_a^2 / \hbar^2$, or, $(\pm p_v')^2 \equiv (\pm \hbar k_d')^2 = \mp (\hbar k_d - q\mathbf{A}_a)(-\hbar k_d - q\mathbf{A}_a) = [\mp(\mathbf{p}_v - q\mathbf{A}_a)]^2$. (B.2) thus generalises to $M^2 c^4 + c^2 p_v'^2 = (\varepsilon - q\phi_a)^2$.

Appendix C. Solution of Dirac equation from the standpoint of particle internal process

We shall here mainly discuss the choice of the solution forms of the Dirac equation from the standpoint of internal processes of the IED particle model for simplicity for spins along z -axis, in an otherwise basically standard procedure. The two equations of (60) or (61) for particle $n = 1$ or 2 are coupled in $\psi_+(n)$ and $\psi_-(n)$ and can not be solved separately as in Appendix B. We need to solve each two, or more generally the four equations of the Dirac equation (60) together. Let the trial functions be: $\psi_\nu(n) = C_{sn} e^{\frac{i}{\hbar}[p_v z - \varepsilon t]}$, $s = +, -, n = 1, 2$. Placing these in (63) and rearranging give

$$\begin{pmatrix} \varepsilon - Mc^2 - q\phi_a & 0 & -[\mathbf{p}_v - q\mathbf{A}_a]c & 0 \\ 0 & \varepsilon - Mc^2 - q\phi_a & 0 & [\mathbf{p}_v - q\mathbf{A}_a]c \\ -[\mathbf{p}_v - q\mathbf{A}_a]c & 0 & \varepsilon + Mc^2 - q\phi_a & 0 \\ 0 & [\mathbf{p}_v - q\mathbf{A}_a]c & 0 & \varepsilon + Mc^2 - q\phi_a \end{pmatrix} \begin{pmatrix} \psi_+(1) \\ \psi_+(2) \\ \psi_-(1) \\ \psi_-(2) \end{pmatrix} = 0 \quad (C.1)$$

(C.1) corresponds to the four linear, homogeneous algebraic equations (C.2) for the $\psi_{s,j}$'s as four unknowns below; for these to have nontrivial solutions, the determinant for the matrix of the coefficients of (C.1) needs be zero. This is $\det = [(\varepsilon - q\phi_a)^2 -$

$M^2c^4]^2 - c^4\mathbf{p}'^4 = 0$, with $\mathbf{p}'_v = \mathbf{p}_v - q\mathbf{A}_a$. This has two degenerate sets of square roots solutions: $\varepsilon - q\phi_a = \pm\sqrt{M^2c^4 + p_v'^2c^2}$, which being identical to (B.2). In view that each particle has internal processes, we thus naturally assign symmetrically two of the four solutions to particle 1, as $\varepsilon - q\phi_a = \pm\sqrt{M^2c^4 + p_v'^2c^2}$, and the other two for particle 2 as $\varepsilon - q\phi_a = \mp\sqrt{M^2c^4 + p_v'^2c^2}$. These two distinct sets of square-roots solutions to the algebraic equation above represent two (distinct, identical) particles, like an electron and a positron, which do not transit from one to the other, a point agreeing with reality and having been stressed by P.A.M. Dirac from the very beginning in [1]. These algebraic solutions are in contrast to the usual problem of eigen values arising generally from boundary conditions and being each possible states of same particle between which transitions generally can occur.

With p_v and ε as known parameters, we further solve the four algebraic equations

$$\begin{aligned} (\varepsilon - (Mc^2 + q\phi_a))\psi_+(1) &= p'_vc\psi_-(1) \quad (a), & (\varepsilon - (Mc^2 + q\phi_a))\psi_+(2) &= -p'_vc\psi_-(2) \quad (b) \\ (\varepsilon + (Mc^2 - q\phi_a))\psi_-(1) &= p'_vc\psi_+(1) \quad (c), & (\varepsilon + (Mc^2 - q\phi_a))\psi_-(2) &= -p'_vc\psi_+(2) \quad (d) \end{aligned} \quad (C.2)$$

corresponding to (C.1), for the wave functions. Taking the imaginary of equations (a) and (b) first, multiplying the resulting equation with equation (c) and the second with (d) respectively on opposite sides, substituting with $\psi_-^*(1)\psi_-(1) = C_{-1}^2$, $\psi_+^*(1)\psi_+(1) = C_{+1}^2$, $\psi_-^*(2)\psi_-(2) = C_{-2}^2$ and $\psi_+^*(2)\psi_+(2) = C_{+2}^2$, we get $(\varepsilon - (Mc^2 + q\phi_a))C_{+1}^2 = (\varepsilon + Mc^2 - q\phi_a)C_{-1}^2$, $(\varepsilon - (Mc^2 + q\phi_a))C_{+2}^2 = (\varepsilon + Mc^2 - q\phi_a)C_{-2}^2$. These have two independent solutions, and in mathematical terms two of the four wave functions can thus be arbitrarily chosen. In view of the IED particle model by which each particle has internal, wave processes consisting of two components in the one-dimensional box, it is natural here that we choose the values for C_{+1} and C_{+2} symmetrically, in the sense also $C_{+2} = -C_{+1}$, with these in the two equations above the values for C_{-1} and C_{-2} then follow to be uniquely given as

$$\begin{aligned} C_{+1} &= \pm \left(\sqrt{\frac{\varepsilon + (Mc^2 - q\phi_a)}{\varepsilon - (Mc^2 + q\phi_a)}} \right)^{1/2} C, & C_{-1} &= \mp \left(\sqrt{\frac{\varepsilon - (Mc^2 + q\phi_a)}{\varepsilon + (Mc^2 - q\phi_a)}} \right)^{1/2} C \end{aligned}$$

where $|C_{+1}C_{-1}| = C^2$, $|C_{+2}C_{-2}| = C^2$. With the above in the trial functions, we get the complete solution for Dirac equation

$$\psi = \begin{pmatrix} \psi_+(1) \\ \psi_+(2) \\ \psi_-(1) \\ \psi_-(2) \end{pmatrix} = \begin{pmatrix} \left(\sqrt{\frac{\varepsilon + (Mc^2 - q\phi_a)}{\varepsilon - (Mc^2 + q\phi_a)}} \right)^{1/2} C e^{i(k_d z - \omega t)} \\ - \left(\sqrt{\frac{\varepsilon + (Mc^2 - q\phi_a)}{\varepsilon - (Mc^2 + q\phi_a)}} \right)^{1/2} C e^{i(k_d z + \omega t)} \\ - \left(\sqrt{\frac{\varepsilon - (Mc^2 + q\phi_a)}{\varepsilon + (Mc^2 - q\phi_a)}} \right)^{1/2} C e^{i(k_d z + \omega t)} \\ \left(\sqrt{\frac{\varepsilon - (Mc^2 + q\phi_a)}{\varepsilon + (Mc^2 - q\phi_a)}} \right)^{1/2} C e^{i(k_d z - \omega t)} \end{pmatrix}. \quad (C.3)$$

Agreeing with the wave functions directly based on IED particle model, $\psi_+(1), \psi_-(1)$ are two opposite travelling component waves of particle 1, and $\psi_+(2), \psi_-(2)$ of particle 2; in the meantime, the spin-up component waves of particles 1 and 2, $\psi_+(1)$ and $\psi_+(2)$ travel in opposite directions and similarly the spin-down component waves.

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